AUTOMATED OPTIMUM DESIGN OF REFRIGERATED WAREHOUSES AND AIR-CONDITIONED BUILDINGS

A Thesis Submitted
In Partial Fulfilment of the Requirements
for the Degree of
DOCTOR OF PHILOSOPHY

by

BHAGWAN DAS GUPTA

to the

DEPARTMENT OF MECHANICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY KANPUR

AUGUST 1977

To

REVEREND DADA JI

CERTIFICATE

This is to certify that the thesis entitled AUTOMATED OPTIMUM DESIGN OF REFRIGERATED WAREHOUSES AND AIR-CONDITIONED BUILDINGS by Bhagwan Das Gupta for the award of the degree of Doctor of Philosophy is a record of bonafide research work carried out by him under my supervision, and has not been submitted elsewhere for a degree.

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CONTENTS

CHAPTER			Page

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS

a	:	Fraction of radiation component available for absorption
A	:	Area
b	:	Transmissivity
B_λ	:	Monochromatic transmissivity
C	:	Specific heat
D	:	Day of the month
$[D]$	:	Matrix of the partial derivatives of the constraints with respect to design variables
e	:	Emissivity of the surface
e'	:	Emissivity of the atmosphere
$\vec{E}$	:	Vector of the partial derivatives of the objective function with respect to design variables
f	:	Objective function
f_{cl}	:	Ratio of surface area of clothed body to the surface area of nude body
F	:	A function of n random variables
F_s	:	Sunlit fraction of the window
g_j	:	j^{th} inequality constraint
h_c	:	Convective heat transfer coefficient
h_{c_i}	:	Convective component of inside film coefficient
h_i	:	Inside film coefficient
h_o	:	Outside film coefficient
h_{r_i}	:	Radiative component of inside film coefficient
H	:	Hour angle

$[H_i]$	:	Symmetric positive definite matrix in variable metric method
$[I]$	:	Identity matrix
I	:	Incident solar radiation
I_d	:	Direct solar radiation
I_D	:	Diffused solar radiation
I_R	:	Reflected solar radiation
I_T	:	Global solar radiation
I_{cl}	:	Thermal resistance of clothing
j	:	Superscript of number of constraints
K	:	Thermal conductivity
k_1, k_2, k_3	:	Constants
K_c	:	Extinction coefficient
l	:	Direction cosine normal to a surface
l	:	Length
L	:	Thickness
L_a	:	Latitude of the place
L_c	:	Cooling load
$\bar{L}_c$	:	Mean value of the cooling load
m	:	Number of constraints
m	:	Mass
m	:	Direction cosine of normal to a surface
M	:	Metabolic rate
n	:	Number of design variables
n	:	Direction cosine of normal to a surface

p	:	Pressure of air
p_j	:	Probability of occurrence
p_a	:	Pressure of water vapour in ambient air
P	:	Principal investment
q	:	Heat rate
Q_g	:	Total heat gain through the glass
r	:	Rate of interest
r_k	:	k^{th} penalty parameter
R	:	Universal gas constant
S_i	:	i^{th} search direction
t_a	:	Air temperature
t_e	:	Solar-air temperature
t_{ei}	:	Solar-air temperature at i^{th} hour
t_{cl}	:	Mean temperature of outer surface of clothed body
t_i	:	Inside air temperature
t_{mrt}	:	Mean radiant temperature
t_{max}	:	Maximum outdoor temperature
t_o	:	Outdoor air temperature
v_i	:	Coefficient of transfer function
V	:	Wind velocity
V_i	:	Volume
w_i	:	Coefficient of transfer function
W	:	Weights
X_i	:	i^{th} design vector

X_j	:	j^{th} design variable
$\underline{\vec{X}}$	:	Mean value of the vector X
$\vec{X}^*$	:	Optimum design point
$\vec{Y}$	:	Vector of design parameters other than X
α_1	:	A parameter in extremal distribution
α^*	:	Minimizing step length
α	:	Thermal diffusivity
δ	:	Declination
ε	:	Small number
η	:	Mechanical efficiency
θ	:	Angle
λ_n	:	Decrement factor
μ	:	Permeability
$\vec{\xi}$	:	Vector of Kuhn-Tucker multipliers
ρ	:	Density
ρ_λ	:	Monochromatic reflectivity
σ	:	Stephen-Boltzmann constant
σ	:	Standard deviation
τ	:	Time
ϕ_n	:	Lag between a harmonic of the solar-air temperature and same harmonic of the inside surface temperature
ϕ	:	Penalty function
ω_n	:	Angular velocity of sinusoidal wave

SYNOPSIS

AUTOMATED OPTIMUM DESIGN OF REFRIGERATED WAREHOUSES AND AIR-CONDITIONED BUILDINGS

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The whole world is striving for the conservation of energy at present. All-round efforts are being made to accomplish any specified task with minimum use of energy. Thus, there is a need to design systems which are both financially feasible and entail minimum functional energy. Most of the thermal system designs are based on empirical relations and tables available in handbooks and other references which do not represent the physical situation truly. The design of refrigerated warehouses and air-conditioned buildings falls in the same category.

In the present context the number of refrigerated warehouses in the country is multiplying fast and consequently the demand on energy is ever increasing. The structural cooling load, which depends on the environment, is quite substantial apart from the commodity cooling load in such buildings. The environmental parameters contributing to the structural

cooling load are probabilistic and uncontrollable. An unified design approach should be thought of for accounting these parameters and reducing the structural cooling load economically. Although some work using graphical method of one dimensional minimization has been reported in the past, so far no attempt has been made in using mathematical programming techniques and accounting for the probabilistic nature of the cooling load.

The aim of this work is to present a unified automated procedure for the optimum design of refrigerated warehouses and air-conditioned buildings. The design problems are formulated as nonlinear mathematical programming problems and solved by using multidimensional optimization techniques.

The thesis has been organized into seven chapters and three appendices. Chapter 1 presents a review of the work done in the area of optimal design of refrigerated and air-conditioned buildings. The scope of minimum energy and minimum total cost designs is discussed.

Some of the multidimensional optimization techniques are stated in Chapter 2. The interior penalty function approach is suggested for transforming a constrained problem into a sequence of unconstrained problems. The Davidon-Fletcher-Powell variable metric method coupled with the cubic interpolation scheme of one dimensional minimization is recommended for solving the sequence of unconstrained optimization problems.

In Chapter 3, the deterministic design of refrigerated warehouses with and without a consideration of internal loads is presented. A definition of the design day based on the hourly outdoor temperature and solar radiations is suggested. Several design problems are solved to illustrate the effectiveness of the method proposed. The minimization of total cost (including equipment, thermal insulation, running and maintenance costs) is taken as the criterion function. Two types of thermal insulating materials and three economics models are considered to evaluate the total cost. The heat gain components are converted into the corresponding cooling load components by using the transfer function method.

In Chapter 4, the design of air-conditioned buildings is considered according to deterministic design philosophy. The problem of design of an office building is solved by taking the total cost and a weighted average of the cooling and heating loads as objectives.

Chapter 5 deals with the probabilistic design of refrigerated warehouses. The probability characteristics of the cooling load are derived by using partial derivative method. Three different problems are solved by taking the mean value of the cooling load and the mean value plus three times standard deviation of the cooling load as the objectives. In one problem the internal surface temperature is not allowed to exceed a prescribed value by more than a specified probability.

The application of extremal distributions to the design of thermal systems is considered in Chapter 6. Three types of extremal distributions are fitted to the hourly maximum temperature and solar radiations. It is found that type III distribution fits the data most closely. The same distribution (type III) is found to fit the yearly maximum temperature data also best. A methodology using the maximum yearly temperature data, is developed for the design of refrigerated warehouses. The results obtained by using extremal distributions and probabilistic design procedure compared well.

In Chapter 7, general conclusions and contributions of this work are summarized along with the recommendations for future research in the field.

The following conclusions are drawn from the present investigation:

(1) It is economical to use thin brick walls with proper insulation thickness to reduce the cooling load while thicker bricks may be desired from strength point of view. Hence a compromise has to be made between cooling load and strength in selecting the brick thickness with proper insulation.

(2) Three different orientations of a given building do not show significant variation in the total optimal cost; it is of the order of 1% while the change in structural load is approximately of the order of 2%.

(3) The optimization results of the problem in which envelope parameters are taken as design variables indicate that, for a given volume, maximum possible height is to be selected not only for ease of mechanical handling of the stored commodity but also for minimizing the total cost. An aspect ratio of approximately 1.5 is found to be economical. The cooling load is reduced by approximately 10% for such a warehouse compared to the one having a lower height with an aspect ratio of 2.0. Same problem is solved with the minimization of total cooling load as the objective and it is found that the load can further be reduced by 8%.

(4) Since the desired inside temperature is higher compared to refrigerated warehouses, the structural load will be less in case of air-conditioned buildings. In these cases it is found that it was not possible to reduce the merit function by more than 3% of the initial value. It is interesting to note that the contribution of the roof to the cooling load is maximum in any building and hence maximum insulation is also desired for the roof. However if one is interested in optimizing the total (cooling and heating) load, then the insulation thickness may not be maximum for the roof.

(5) Although the probabilistic design approach required more computational effort, it represents a more rational and realistic procedure for the design of refrigerated and air-conditioned systems. The problem with probabilistic

constraint demonstrates a procedure by which it is possible to control the inside surface temperature at any desired value which may be required in specific cases.

(6) In the design example using extremal distributions, it is found that the optimal load value is about 4% higher than in the case of probabilistic design. This shows that the designs based on extremal distributions are more conservative.

(7) Finally the interior penalty function method of optimization worked fairly well with all the problems considered and is recommended for the optimum design of thermal systems.

The contributions of the present work can be stated as:

(1) Development of an automated optimization procedure for the design of refrigerated warehouses and air-conditioned buildings,

(2) Application of statistical meteorological data in the optimum design of such systems,

(3) Demonstration of the use of extremal distributions in the field of thermal systems design, and

(4) Development of generalized computer programs for the optimum design of refrigerated warehouses and air-conditioned buildings.

CHAPTER 1

INTRODUCTION

The ultimate aim of any engineer engaged in the design of thermal systems is to produce a financially feasible and economically viable design which achieves a desired performance level. In the past, when energy costs were low and its availability potential was high, the energy required for the functioning of any system or the running cost over its lifetime was of secondary importance. But now with limited energy resources and high costs, it is necessary to conserve energy or use it optimally. A system should be designed for minimum functional energy without affecting its performance. In the case of refrigerated warehouses or air-conditioned buildings, the energy requirements can be reduced by suitably designing them. Generally, systems with high initial costs (involving sophisticated design concepts) are presumed to have lower running costs with high levels of performance, but such systems may not be always economically feasible.

The need for refrigerated warehouses and air-conditioned buildings is increasingly felt in the country. This results in the demand of more and more functional energy. Thus there is a necessity for improving the design of such systems without affecting the total cost much. Generally a number of alternatives can be thought of for designing a new system or

improving an existing one. In refrigerated warehouses the structural cooling load, which is dependent on environment, is quite substantial apart from the commodity cooling load. The environmental parameters contributing to the structural cooling load are probabilistic and uncontrollable. An unified automated optimum design procedure for refrigerated warehouses and air-conditioned buildings is suggested in the present work. A method of considering the probabilistic nature of the input parameters in the formulation is also presented. The constraints considered in the probabilistic formulation are converted into their equivalent deterministic form by making use of probability principles. In this work, the systems considered for design are confined to buildings only. The design of equipment needed to perform the desired function is not considered.

1.1 Review of Literature

The work available in the direction of design of refrigeration and air-conditioning systems is summarized in the following paragraphs.

1.1.1 Analysis of cooling and heating systems

In the analysis of cooling or heating systems the designer is interested in computing the heating or cooling load exactly. The parameters which influence these loads are the ambient air temperature, direct and diffused solar radiations,

air velocity, characteristic of the enclosure walls and the orientation of the enclosure. Three methods have been developed for accounting the periodicity of external conditions:
i) Threlkeld's classical approach [1,2,3,4] (ii) Transfer function method [5] and (iii) Finite difference method [6]. A computer programme has been developed by Lokmanhekim and Henninger [7] to evaluate the heating or cooling load as well as the energy requirement for the enclosure. The programme, in addition, may be used for equipment sizing, system simulation and economic analysis to provide the desired inside conditions. The development of another computer programme, called Gate E, which consists of a set of basic programmes in series, is reported in references [8,9]. Buffington [10] has presented transmission matrix method for the computation of heat gain through exterior walls and roof when the outside and inside air temperatures are transient. An elaborate design analysis of these types is desirable and has become necessary over the years due to the ever increasing shortage of energy, rising building construction costs and enormous capital and operating costs involved in air-conditioning and refrigerating system equipment.

The finite difference method [6] uses an iterative procedure for computing the heat transfer through walls. However, this technique is not very efficient for composite complex structures. In the present work the Threlkeld's

classical approach [1,2,3,4] has been used to compute the heat gain due to walls and roof, although this method gives a little conservative values of the loads.

The hourly dry bulb temperature and direct and diffused solar radiations data is collected for two cities in India [11, 12,13]. The mean and standard deviations of the above parameters and solar-air temperature are computed for all the 365 days of the year. This information is further used to select a design day for each month. A definition based on the concept of solar-air temperature is used to define a design day. Carrier [14] gave a definition which accounts for the solar radiation and outdoor temperatures separately. A general computer programme using the hourly meteorological data is developed to compute the heat gain using the classical approach of Threlkeld [1]. In addition to the load due to transient conduction through the walls, the programme also evaluates (a) heat gain through fenestrations [15] (b) heat gain caused by the diffusion of moisture through permeable building materials [15], and (c) heat generation within the conditioned space that may be due to [15] (i) lighting [14,15,16], (ii) human occupation and activities [14,15], (iii) electric motors [14,15], (iv) appliances [14,15], (v) one or more of the processes associated with products such as chilling and cooling to the storage temperature [14,15,17], (vi) respiration heat released from the stored products [15,18] and (vii) infiltration and ventilation

heat gain [14,15,19]. The calculations are carried out by using the formulae specified in the cited references and finally, the instantaneous heat gain components are transformed to the corresponding cooling load components by using transfer function method [20].

Green and Smith [21] have identified the significant role of probability theory in the design of building systems and associated different probabilities to different events of failure. In the present work, a probabilistic optimum design procedure is presented by considering the randomness of the input parameters. The probabilistic parameters needed for evaluating the cooling or heating load are evaluated by using the approximate partial derivative method [22].

Three types of extremal distributions [23,24] are fitted to the daily maximum values of temperature and global solar radiation for two cities in India and the best fitted distribution is used to generate the additional information needed for the design. A problem is formulated and solved to show the application of extremal distributions in the optimum design of refrigeration and air-conditioning systems.

1.1.2 Optimization

With limited availability of useful energy and consequent increases in costs, the designers are made to think for the optimal use of energy. Serious steps are being taken

in this direction through ASHRAE standard 90-75 on energy conservation in new building design [25]. Some work has been initiated in this direction even earlier. McClure [26] and Stephen [27] considered the design of building systems to reduce the energy requirements. Bonar [28] has analysed the different parameters and factors that affect the economics of a refrigeration system for a warehouse with reference to the minimization of energy costs.

Stephen [29] has discussed about the economic insulation thickness while Spielvogel [30] has found that more insulation can increase the energy consumption. Tobias [31] has also discussed some methods for reducing energy requirements in residential buildings. Field [32] has given the design of a window for minimum energy requirements. Rudoy and Fernando [33] have discussed the effect of building envelope parameters on energy requirements. Tseng-Yao [34] has described qualitatively the effect of envelope parameters on energy requirements, while Ambrose [35] has studied the architectural aspects in relation to energy conservation. Joseph [36] and Ross [37] have discussed the recent trends and energy saving procedure in the design of refrigerated warehouses. Henize [38] has compared the costs of installation and exploitation of high rack construction with those of single storied refrigerated warehouses. Wood and Donoso [39] have discussed about energy conversion efficiencies in different buildings. Stoecker [40] has

discussed the different economics models for investment while Coad [41] has given a methodology for the life cycle cost analysis. The first systematic approach for the optimum design of air-conditioned systems was presented by Wilson and Templeman [42] using geometric programming technique. Jog [43] has suggested some structural improvements of cold-storage buildings.

1.2 Objective and Scope of the Present Work

It can be observed from the available literature that the economics part of the design of refrigerated warehouses and air-conditioned buildings has not been systematically studied so far. Only Wilson and Templeman [42] have made an attempt in this direction using geometric programming method. This technique (of geometric programming) can only be employed with a special class of optimization problems. The one dimensional minimization (by graphical method) to obtain the economical insulation thickness has been studied by a number of investigators. However, mathematical programming techniques have not been employed to obtain the solution of a general problem so far. Also, the design criterion used in conventional designs is very much conservative and is based on the peak load value. However, this value may or may not occur even once in a year. Hence a more comprehensive definition of the design day over a month is used in this work. A weighted average of the monthly loads is taken as the yearly load.

one to minimize the total cost of the system including the initial, running and maintenance costs as well as insulation costs and other to minimize the total energy requirements. Two economic models i.e. i) Present-worth method and ii) Annual cost method are considered to compute the total cost of the system. The annual cost model is finally used as the present worth method has the inherent drawback of introducing complications when the two prospective investments have different lives. However, Coad [41] has preferred to use life cycle cost analysis for the optimum investment policy.

In all the above designs the optimum points are further analysed for sensitivity by changing each component of the optimum design vector gradually. This study facilitates the designer in knowing the parameters that are more important and also in knowing the effect of choosing near optimal designs. Further, a knowledge of insensitive parameters can be used in reducing the number of design variables in subsequent designs.

1.3 Organization of the Thesis

The thesis has been organized into seven chapters and three appendices. Chapter 1 presents a review of the work done in the area of optimal design of refrigeration and air-conditioning systems. Some initiative has been taken by the ASHRAE task group in finding ways of reducing the energy requirements in new building designs. It is becoming conventional now to go for

higher and higher heights of the warehouses which help in the mechanization needed for the movement and transportation of the commodity. Certain values of overall heat transfer coefficients are suggested to reduce the energy requirements; but how to obtain these values economically with the existing materials still remains a problem.

Some of the efficient multidimensional optimization techniques are stated in Chapter 2. These techniques are well developed and have been used in the fields of structural and mechanical design. The potentialities of these optimization methods have not been fully realized in the field of thermal systems design. In the present work, the interior penalty function method [44] is used for solving both deterministic and probabilistic optimization problems. The Davidon-Fletcher-Powell variable metric method [45] coupled with the cubic interpolation scheme [46] of one dimensional minimization is used to solve the resulting sequence of unconstrained optimization problems. Although other methods like Zountendijk's method [47] of usable feasible directions and gradient projection method [48,49] are also available, the interior penalty function technique has been selected for the present application due to its generality and reliability.

The deterministic design of refrigerated warehouses is considered in Chapter 3. In the present work, the hourly cooling load is computed by using the classical approach of

Threlkeld [1]. A definition based on the concept of solar-air temperature is used for the design day and the total meteorological data is processed to select the design day of each month. A weighted average of monthly loads is taken as the average load over the year. It is assumed that the plant runs twenty four hours a day to maintain the desired inside conditions. The objective for minimization is taken as the total cost which includes the initial, running and maintenance costs of the plant and the insulation cost. Two one dimensional problems, are involving only a roof and the other involving only a wall, are solved. In the first problem, two types of insulation, namely, mineral wool and expanded polystyrene are used and it is found that mineral wool is economical compared to expanded polystyrene. Numerical results for the second problem are obtained by considering three different brick thicknesses for the walls and it is observed that thinner walls with proper insulation are economical. A two variable problem is solved to optimize the outside surface absorptance for minimum cooling load. The effect of orientation on the optimum design of a warehouse (with four walls and roof) is also studied by considering three different orientations. A more general problem of the design of a warehouse is considered by including the envelope parameters as design variables along with the insulation thicknesses of walls and roof. The general warehouse design problem is also solved with the minimization of the total cooling load as

the objective and the two sets of results are compared with each other from financial and energy considerations.

Chapter 4 deals with the automated optimum design of air-conditioned buildings according to deterministic design philosophy. Here the inputs to the system are the same as in the case of refrigerated warehouses but the requirements are more rigorous and the system itself may be subjected to a wide variation in the cooling load. The inside temperature and humidity should be such that most of the personnel inside the building feel comfortable throughout the year. This may need sometimes cooling and sometimes heating of the building. A merit function which takes care of both cooling and heating loads, is constructed and the problem is solved for the minimization of total cost. The same problem is solved for the minimization of total load and the results of the two problems are found to be comparable.

In the Chapter 5, the probabilistic optimum design of refrigerated warehouses is considered. Here the design day is defined and selected on the basis of mean plus three times the standard deviation of solar-air temperature. The expressions for mean and standard deviation of cooling load are derived by using an approximate method [22] to avoid complicated analysis procedure. Three problems are solved to demonstrate the effectiveness of the probabilistic design procedure presented. In the first problem, the merit function is taken as the mean

value of the cooling load and in the second it is chosen as the mean value plus three standard deviations of the cooling load. In the third problem the mean value of the cooling load is minimized subjected to a limitation on the inside maximum surface temperature. In this case, since the constraint itself is probabilistic, it is converted into an equivalent deterministic constraint before applying the mathematical programming techniques. This type of restriction has a good physical significance if the stored commodity is very sensitive to temperature and heat radiation. In fact since the input parameters are probabilistic, this type of design approach is more rational than the deterministic one.

The application of extremal distributions [24] in the design of thermal systems is considered in the Chapter 6. In the past extremal distributions have been used in hydrological and meteorological problems like floods, droughts, gusts, etc. [23]. In a limited sense, these distributions have also been used in mechanical design to describe the strength of metals in fatigue. They have not been applied in the design of thermal systems so far. Three types of distributions for the maximum value of outside daily temperature and global solar radiation are tried for two different places in India. The same distributions are fitted to the yearly maximum temperature. These types of distributions can also forecast the maximum or minimum values at any particular time and some of these forecasts have been found to be quite satisfactory. An optimum design

procedure using extreme value distributions is presented and is illustrated with the problem of design of a refrigerated warehouse.

In Chapter 7, general conclusions and contributions of this work are summarized along with recommendations for future research in this field.

Appendix A presents the method of calculating the heat gain by the classical approach of Threlkeld [1]. Appendix B gives the data (in tabular form) used for the computation of heat gain through different sources, while Appendix C gives the tables of coefficients required for conversion of heat gain to corresponding cooling load component.

CHAPTER 2

OPTIMIZATION PROCEDURE

Any problem in the area of thermal system design can be formulated and solved according to either deterministic or probabilistic design philosophy. If all the parameters affecting the design problem are deterministic, the problem can be solved according to the deterministic design philosophy. On the other hand, if some of the design parameters are random in nature, the problem has to be solved according to the probabilistic design philosophy. If limitations on the performance and other external constraints on the design can be stated and a basis for choosing between alternate acceptable designs can be established, it is possible to pose the design problem in the form of a mathematical programming problem.

In the deterministic design philosophy a general mathematical programming problem can be stated as follows:

$$
\begin{aligned}
&\text{Find } \vec{X} \\
&\text{such that } f(\vec{X}) \text{ is minimum} \\
&\text{subjected to the constraints} \\
&\qquad g_j(\vec{X}) \leq 0 \qquad j = 1,\ldots,m
\end{aligned}
\tag{2.1}
$$

In the probabilistic design philosophy, however, a general mathematical programming problem can be stated as:

Find $\vec{X}$

which minimizes a multivariable function $f(\vec{X}, \vec{Y})$

subjected to the probabilistic constraints (2.2)

$$P[g_j(\vec{X}, \vec{Y}) \leq 0] \geq p_j, \qquad j = 1,\ldots,m$$

where $\vec{X}$ is the vector of design variables and $\vec{Y}$ is the vector of other parameters affecting the design problem. Here the components of $\vec{X}$ and $\vec{Y}$ are assumed to be random variables, f represents the objective function and $P[g_j(\vec{X}, \vec{Y}) \leq 0] \geq p_j$ denotes that the j^{th} constraint has to be satisfied with a probability of greater than or equal to some specified quantity, p_j, $j = 1,\ldots,m$, where $0 \leq p_j \leq 1$.

Since f and g_j are functions of the random variables $\vec{X}$ and $\vec{Y}$, f and g_j will also be probabilistic in nature. In actual computations, the problem stated in Eq. (2.2) is converted into an equivalent deterministic problem to facilitate its solution. For this the deterministic objective function is taken as either the mean value of f or the mean value plus constant number of standard deviations of f. The probabilistic constraints are also converted into deterministic form by making use of probability principles.

2.1 Choice of the Solution Method

The three general classes of widely used nonlinear programming methods are as follows:

i) Gradient projection method of Rosen [48] which was subsequently modified by Goldfarb [49]: Though this method works well with linear constraints, its efficiency is considerably reduced in the case of nonlinear constraints.

ii) Feasible direction method of Zountendijk [47]: This method is based on the generation of usable feasible directions at constraint boundaries. Although this method works in a direct manner in solving the problem, the analyses during optimization have to be done accurately as they influence the rate of convergence and accuracy during optimization.

iii) Penalty function methods: In these methods, the constrained formulation is transformed into a sequence of unconstrained optimizations which can be solved without much difficulty. These methods are quite reliable and their sequential nature allows a gradual approach to criticality of constraints. These methods allow coarse approximations to be used during early stages of optimization procedure and finer or more accurate approximations during the final stages. In the present work, the penalty function method of Fiacco and McCormick [44] is used as it has been found to be quite reliable.

The basic optimization problem of the form (2.1) can be converted into an unconstrained minimization problem by constructing a function of the form

$$\phi(\vec{X}, r_k) = f(\vec{X}) + r_k \sum_{j=1}^{m} G[g_j(\vec{X})] \qquad (2.3)$$

where G is some function of the constraints. If the minimization of the ϕ-function is repeated for a sequence of values of the response factor r_k, the solution may be brought to converge to that of the original constrained problem given by Eq. (2.1).

2.2 Fiacco McCormick Interior Penalty Function Method [44]

In this method, the objective function is augmented with a penalty term consisting of the constraints as shown below:

$$\phi(\vec{X}, r_k) = f(\vec{X}) - r_k \sum_{j=1}^{m} \frac{1}{g_j(\vec{X})} \qquad (2.4)$$

where $\phi(\vec{X}, r_k)$ is called the penalty function, and r_k is called the response or penalty parameter in k^{th} minimization step. The minimization of ϕ-function is performed for a decreasing sequence of r_k so that

$$r_{k+1} < r_k \qquad (2.5)$$

The minimization of $\phi(\vec{X}, r_k)$ requires a feasible starting point and, in the present work, it is found by trial and error. Since each of the designs generated during the optimization process lies inside the acceptable design space, the method is classified as interior penalty function formulation.

2.3 Davidon-Fletcher-Powell Variable Metric Unconstrained Minimization Method

In the penalty function formulation, since a sequence of unconstrained minimization has to be performed, the selection of an efficient method of unconstrained minimization becomes very important. All the methods of unconstrained minimization find a sequence of improved approximations to the optimum according to the iteration

$$\vec{X}_{i+1} = \vec{X}_i + \alpha^* \vec{S}_i \tag{2.6}$$

where $\vec{X}_{i+1}$ is the design vector corresponding to the minimum of the $\emptyset$-function along the current direction $\vec{S}_i$, $\vec{X}_i$ is the starting design vector and α^* is the minimizing step length in the direction $\vec{S}_i$. There are several methods available for finding the search direction $\vec{S}_i$ in Eq. (2.6).

In the present work, the Davidon-Fletcher-Powell variable metric method [45] is used. This method can be considered as a quasi-Newton algorithm, and is a very powerful general procedure for finding a local unconstrained minimum of a function of many variables [50]. In this method, the i^{th} search vector $\vec{S}_i$ in Eq. (2.6) is computed as follows

$$\vec{S}_i = - [H_i] \cdot \nabla\emptyset(\vec{X}_i) \tag{2.7}$$

where $\nabla\emptyset(\vec{X}_i)$ denotes the gradient of the $\emptyset$-function at $\vec{X}_i$ and $[H_i]$ is a positive definite symmetric matrix. The matrix $[H_i]$ is updated according to the following procedure:

$$[H_{i+1}] = [H_i] + [M_i] + [N_i] \tag{2.8}$$

where

$$[M_i] = \frac{\alpha^* \vec{S}_i \vec{S}_i^T}{\vec{S}_i^T \vec{V}_i} \tag{2.9}$$

$$[N_i] = -\frac{[H_i] \vec{V}_i \vec{V}_i^T [H_i]^T}{\vec{V}_i^T [H_i] \vec{V}_i} \tag{2.10}$$

and $$\vec{V}_i = \nabla\phi(\vec{X}_{i+1}) - \nabla\phi(\vec{X}_i) \tag{2.11}$$

The updating of $[H_i]$ preserves the symmetric positive definiteness of $[H_{i+1}]$ which ensures the stability of the procedure. The positive definiteness of $[H_{i+1}]$ is influenced only by the accuracy with which α^* is determined. To start with $[H_0]$ is taken as identity matrix.

2.4 One Dimensional Minimization Method

In the present work, the one dimensional minimization is accomplished by cubic interpolation scheme [46]. In this method, the algorithm used to compute α^* is reapplied until the cosines of the angle between the vectors $\vec{S}$ and $\nabla\phi$ at the minimizing step length α^* is sufficiently small, i.e.

$$\cos\theta = \frac{\vec{S}_i^T \cdot \nabla\phi_{i+1}}{|\vec{S}_i| \cdot |\nabla\phi_{i+1}|} < \varepsilon \tag{2.12}$$

where ϵ is a small number whose value is taken as 0.05 in the present work. This ensures that $\vec{X}_{i+1}$ is minimum along the direction $\vec{S}_i$. However, to reduce the computer time, the number of cubic interpolations is limited to three.

2.5 Additional Considerations and Convergence Criteria

i) Starting point $\vec{X}_o$

For the minimization of $\phi(\vec{X}, r_1)$, the starting feasible point $\vec{X}_o$ is found by a process of trial and error. Each subsequent stage used the solution of the previous stage as a starting point. In some cases the overall procedure may be accelerated by employing an extrapolation technique [51] to determine starting points for subsequent unconstrained minimization cycle. The starting point so obtained must be checked to ensure that they satisfy the constraints, because at early stage, it is necessary to start the unconstrained minimization of $\phi(\vec{X}, r_k)$ from an acceptable design point.

ii) Initial value of r_k

If r is large, the function is easy to minimize, but the minimum may be far from the desired solution to the original constrained minimization problem. On the other hand, if r is small the function will be hard to minimize. In the present work the value of r_1 is chosen such that

$$1.5f(\vec{X}_o) \leq \emptyset(\vec{X}_o, r_1) \leq 2.0f(\vec{X}_o) \tag{2.13}$$

iii) Subsequent values of r_k

The total number of r's to be employed is given as an input to the problem and the values of r_{k+1} are found by using the ratio

$$\frac{r_{k+1}}{r_k} = 0.1 \tag{2.14}$$

iv) Restarting the [H] matrix

In the case of highly distorted or eccentric functions, it might occur after few iterations that $\vec{S}_i^T \cdot \nabla\emptyset_i$ is positive, indicating that $\vec{S}_i$ is not a direction of descent. When this happens, the most efficacious remedy is to set $[H_{i+1}]$ back to $[H_o]$ and proceed as if starting over again.

v) Termination of minimization for each r_k

For each r_k, the minimization of the $\emptyset$ function is terminated when the predicted percentage difference between the current and the optimal $\emptyset$-values is less than a small quantity, that is,

$$\delta = \frac{\nabla\emptyset_i^T [H_i] \nabla\emptyset_i}{2\emptyset_i} = \frac{\vec{S}_i^T \nabla\emptyset_i}{2\emptyset_i} < \varepsilon \tag{2.15}$$

where ε is a small quantity. The value of ε used in the present work varied from about 0.1 for r_k to 0.0005 for higher r_k.

vi) Gradient of $\emptyset$-function

The gradient of $\emptyset$-function can be expressed as

$$\nabla\emptyset = \nabla f + r \sum_{j=1}^{m} \frac{1}{g_j^2} \cdot \nabla g_j \tag{2.16}$$

However, the gradient of $\emptyset$-function is computed by using forward difference technique in the present work.

vii) Number of cubic interpolations

A maximum of five cubic interpolations are allowed in each one dimensional minimization. Out of these, only the final interpolation involves the evaluation of the gradient $\nabla\emptyset$. All preliminary interpolations use a perturbation scheme to determine the dot product $\vec{S}^T \cdot \nabla\emptyset$ as

$$\vec{S}^T \cdot \nabla\emptyset \approx \frac{\emptyset(\alpha^+) - \emptyset(\alpha^-)}{(\alpha^+ - \alpha^-)} \tag{2.17}$$

viii) Kuhn-Tucker conditions

An optimum solution $\vec{X}^*$ is characterized by the property that there are not other feasible designs $\vec{X}$ in the immediate neighbourhood of $\vec{X}^*$ which corresponds to a lower value of the objective function than $f(\vec{X}^*)$. Mathematically this can be expressed by the Kuhn-Tucker conditions

$$\frac{\partial f}{\partial X_i} + \sum_{j \in J} \xi_j \frac{\partial g_j}{\partial X_i} = 0 \qquad i = 1,2,\ldots,n \tag{2.18}$$

$$\xi_j \geq 0 , \qquad j \in J \tag{2.19}$$

where ξ_j are called the Kuhn-Tucker multipliers and J is the set of indices of active constraints. Kuhn-Tucker conditions are necessary and sufficient for a global minimum only in the case of convex programming problems. However, they can be used as the necessary conditions to test the minimum of any practical design problem. In the present work the Kuhn-Tucker conditions are checked at the optimum design point.

ix) Relative minima

In order to see whether any relative minima exist in the design space, two different starting points may be used for the sequence of minimization for any example. If the two sequences lead to the same optimum design (except for small differences that might occur due to numerical instability), it can be assumed that local optimum is same as the global optimum.

x) Reducing the total computational time

It has been observed that some of the automated optimum design problems take a longer time to satisfy the prescribed convergence criteria even after reaching very near to the optimum design point. This happens whenever

the function being minimized is highly distorted. In such cases, it may not be worthwhile to try to reach the exact minimum to obtain about 0.5 or 1% decrease in the objective at the cost of 40 to 50% more computation time. This type of situation can be identified by inspecting the progress of the optimization path at various stages.

CHAPTER 3

FORMULATION AND SOLUTION OF DETERMINISTIC DESIGN PROBLEM

Cooling or heating systems are designed to maintain the desired indoor conditions. The outdoor conditions will be quite different from the desired indoor conditions. The outdoor temperature, solar radiation, air velocity and humidity are beyond the control of the designer while the indoor conditions are dictated by the nature of the utility for which the system is designed. Generally, a refrigerated warehouse, for example, is desired for long term storage of frozen or perishable commodities at a selected low temperature and high relative humidity. In such cases the task of the designer becomes extremely difficult in terms of computing optimal design parameters which take full care of the uncontrollable outside conditions and achieve the desired inside conditions.

3.1 Solar-Air Temperature and Design Day

The first problem of the designer is, to choose the outside conditions such that the system so designed does not fail and at the same time will not be very conservative. To obtain the outdoor design conditions, the hourly dry bulb temperature, global and diffused solar radiations and wind velocity are to be considered at the specified location. Since there is a large

variation in the magnitude of these variables over each day of a year, the problem is to decide the values of these variables based on which the system is to be designed. The general practice is to take extreme values of these variables that occurred over a span of years, which are known as design values. But a design based on these values will be extremely conservative because such design conditions may not repeat even once in the total life of the system. In the Carrier handbook of air-conditioning system design [14], a design day is defined as the day

(a) On which the dry and wet bulb temperatures are peaking simultaneously,

(b) When there is a little or no haze in the air to reduce the solar heat, and

(c) On which all the internal loads are normal.

Design based even on the criterion of design day will be over safe. Hence a concept based on a combination of temperature and solar radiation called solar air temperature, which will cause the same rate of heat flow and temperature distribution through the material as in the actual situation, is used in the present work. Mathematically the solar-air temperature (t_e) can be

$$t_e = t_o + (a\,I/h_o) + [e\sigma T_o^4(e'-1)/h_o] \qquad (3.1)$$

where t_o = outdoor air temperature

T_o = absolute outdoor air temperature

a = absorptivity of the surface for solar radiation

e = emissivity of the surface

e' = emissivity of the atmosphere

σ = Stephen-Boltzmann constant

h_o = outside surface heat transfer coefficient

I = total solar radiation (direct and diffuse) incident on the surface.

Usually the third term on the right hand side of Eq. (3.1) is ignored while calculating the solar-air temperature as its contribution is very small compared to the other two terms. Based on the concept of solar-air temperature, a critical day is defined in each month as the day on which the average solar-air temperature is maximum during that particular month and all internal loads remain normal. Twelve such critical days are chosen over a year. Thus it is not a single value of the temperature that dictates the design but the temperature, solar radiation, air velocity and humidity data over twelve critical days form the basis for computing the cooling or heating load. The computed values of solar-air temperature on the critical days are given in Table 3.1 for New Delhi, India. In reality, the maximum outside conditions may affect the load after few hours of their occurrence depending on the thermal lag of the building. By then, the other conditions like internal loads may be such that the total load may not be at the peak. Further, the peaks occur only for short durations. Hence a weighted

average of the cooling load over the twelve critical days is obtained, which will be more realistic and safe.

3.2 Heat Gain, Cooling Load and Heat Extraction Rate [52]

The terms heat gain, cooling load and heat extraction rate as used in the design of cooling and/or heating systems are defined as follows.

3.2.1 Heat gain

Instantaneous rate of heat gain is the rate at which heat enters into or is generated within a space at a given instant of time. It is classified in two ways: (i) according to the way it enters the space and (ii) according to whether it is sensible or latent gain. The first classification is necessary to distinguish heat gains due to different modes of heat transfer and to calculate energy transfer using appropriate equations. The second classification is important for proper selection of cooling equipment.

3.2.2 Cooling load

Cooling load is the rate at which heat must be removed from the space to maintain the desired room air temperature. It is extremely important to note that the summation of all heat gains at any time does not necessarily equal to the cooling load of the space at that instant. The heat gain due to

radiation is partially absorbed by the surfaces and contents of the space and is not felt by the room air until sometime later.

3.2.3 Heat extraction rate

It is the rate at which heat is removed from the conditioned space. This rate will be equal to the cooling load only when room air temperature is kept constant which rarely happens. In most of the cases the control system, in conjunction with the intermittent operation of the cooling equipment, will cause a swing in room temperature. Therefore, the computation of heat extraction rate would give a more realistic value of the energy removal by the cooling equipment than just the instantaneous value of the cooling load.

3.3 Computation of Heat Gain

The heat gain from various components of a system can be classified as (i) heat gain through exterior walls, roof and floor and (ii) heat gain through different sources inside the conditioned space. It is evaluated as follows.

3.3.1 Heat gain through exterior walls and roof

Heat transfer through walls and roof is computed by the classical approach of Threlkeld [1] given in Appendix A for a homogeneous wall. The analysis is extended to composite walls by replacing the composite wall by an equivalent homogeneous wall.

3.3.2 Analysis for composite wall

The relations for heat gain are given in Appendix A for a homogeneous wall. Mackay and Wright [4] have proposed the following conversion equations to reduce a composite wall to an equivalent homogeneous wall. The ratio of thickness (L) to conductivity (K) of the equivalent wall in terms of its values for individual layers is given by

$$\left(\frac{L}{K}\right)_{eq} = \left(\frac{L}{K}\right)_{i} + \left(\frac{L}{K}\right)_{o} + \left(\frac{L}{K}\right)_{m1} + \left(\frac{L}{K}\right)_{m2} + \ldots \qquad (3.2)$$

where the subscript eq indicates equivalent wall, and i, o, m1, m2,... denote inside, outside, first middle, second middle,... layer of the compositive wall respectively.

Similarly the product $(K\rho C)$ for the equivalent homogeneous wall is given by

$$(K\rho C)_{eq} = \frac{1}{\left(\frac{L}{K}\right)_{eq}} \left[1.1\left(\frac{L}{K}\right)_{i} (K\rho C)_{i} + 1.1\left(\frac{L}{K}\right)_{m1} (K\rho C)_{m1} + 1.1\left(\frac{L}{K}\right)_{m2} (K\rho C)_{m2} + \ldots \right] + \frac{(K\rho C)_{o}}{\left(\frac{L}{K}\right)_{eq}} \left[\left(\frac{L}{K}\right)_{o} - 0.1\left(\frac{L}{K}\right)_{m1} - 0.1\left(\frac{L}{K}\right)_{m2} - \ldots \right] \qquad (3.3)$$

where ρ is the density,

and C is the specific heat of the material of different layers of the wall.

The products S_nL and S_nK can be evaluated for the composite wall using Eq. (A.11) of Appendix A:

$$S_nL = \left(\frac{\omega n}{2\alpha} L^2\right)^{1/2} \tag{3.4}$$

Substituting $\alpha = \frac{K}{\rho C}$ in Eq. (3.4),

$$S_nL = \left(\frac{\omega n}{2K} \rho C L^2\right)^{\frac{1}{2}} = \left(\frac{\omega n}{2} \cdot \frac{L^2}{K^2} \cdot K\rho C\right)^{1/2} \tag{3.5}$$

Equation (3.5) for an equivalent homogeneous wall can be written as

$$(S_nL)_{eq} = \left(\frac{\omega n}{2} \quad (K\rho C)_{eq}\right)^{\frac{1}{2}} \left(\frac{L}{K}\right)_{eq} \tag{3.6}$$

Substituting the values of $\left(\frac{L}{K}\right)_{eq}$ and $(K\rho C)_{eq}$ from Eqs. (3.2) and (3.3) into Eq. (3.6), the expression for $(S_nL)_{eq}$ can be obtained. Similarly since

$$(S_nK) = \left(\frac{\omega n}{2\alpha} K^2\right)^{1/2} \tag{3.7}$$

$(S_nK)_{eq}$ can be evaluated as

$$(S_nK)_{eq} = \left(\frac{\omega n}{2\alpha} (K\rho C)_{eq}\right)^{1/2} \tag{3.8}$$

The values of $\left(\frac{L}{K}\right)_{eq}$, $(K\rho C)_{eq}$, $(S_nL)_{eq}$ and $(S_nK)_{eq}$ are to be substituted in place of their corresponding quantities in the relations given in Appendix A to evaluate the heat gain through the composite wall.

3.3.3 Solar heat gain

The incident solar radiation (I) needed for the computation of solar-air temperature in Eq. (A.4) of Appendix A calculated by using the hourly data of global and diffused solar radiations [12] according to the following procedure:

Known data:

Latitude of the place = L_a .

Direction cosines of normal to the surface = (l,m,n)

Global radiation = I_T

Diffused radiation = I_D

The values of declination (δ) and equation of time (EQT) needed in this computation are given for 21^{st} day of each month in reference [15]. The values for the other days of the month are linearly interpolated in the present work. The values of declination and equation of time for 21^{st} day of each month are given in Tables B.1 and B.2 of Appendix B.

The hour angle is given by

$$H = 0.25 \times MIN \tag{3.9}$$

where MIN = time in minutes from solar noon. It is obtained by adding the value of equation of time EQT corresponding to the day D of the month to the local time. Then this time is to be evaluated with reference to solar noon in minutes.

Direction cosines of the solar beam are given by

$$\cos Z = \sin L_a \sin \delta + \cos L_a \cos \delta \cos H \tag{3.10}$$

$$\cos W = \cos \delta \sin H \tag{3.11}$$

$$\cos S = \pm (1 - \cos^2 W - \cos^2 Z)^{1/2} \tag{3.12}$$

where $\cos S > 0$, if $\cos H > \tan\delta / \tan L_a$.

The cosine of the incident angle θ is given by

$$\cos\theta = l \cos Z + m \cos W + n \cos \delta \tag{3.13}$$

Total solar radiation (I) incident on the wall per unit area is given by

$$I = (I_T - I_D) \cos\theta + I_D \tag{3.14}$$

For horizontal surfaces the values of I_D are taken same as the values reported by meteorological centres [12] while for vertical surfaces they are assumed to be equal to half the values given for horizontal surfaces [53]. The value of heat gain due to solar radiation (I) so obtained is used further in the calculation of heat gain from exterior walls and roof.

3.3.4 Effect of corners

All the warehouses and air-conditioned buildings are three dimensional structures. The exact amount of heat gain in such structures can be computed if the effects of corners, depressions in walls and other departures from one dimensional

systems are considered. As one dimensional approximation neglects the energy conducted through the corners, any estimate of heat transfer based on one dimensional approximation and inside area of the wall underestimates the true heat gain at any given instant. On the other hand, if buildings are treated as one dimensional structures and the external wall surface area is used in computing the conducted heat energy then the true energy conducted will be overestimated. In fact, a two dimensional transient conduction equation should be considered to account for the corner effects. Jacob and Hawkins [54] have given some shape factors to this effect in steady state heat conduction. In the present work the external surface area is considered for the computation of heat gain through walls, as the solution of two dimensional transient equation is complicated and the shape factors are available only for steady state.

3.3.5 Outside and inside film coefficients

The outside and inside film coefficients needed in the computation of heat gain are calculated by using the following equations [6].

Outside film coefficient: The outside film coefficient, h_o, depends on the wind velocity and the type of surface. For different types of surfaces the equations for h_o are given below:

(i) For very smooth surfaces like glass

$$h_o = 3.41 + 0.85\ V \tag{3.15}$$

(ii) For smooth surfaces such as planed wood and plaster

$$h_o = 4.4 + 0.91\ V \tag{3.16}$$

(iii) For moderately rough surfaces like finished concrete

$$h_o = 6.34 + 1.21\ V \tag{3.17}$$

(iv) For rough surfaces such as stucco

$$h_o = 6.84 + 1.52\ V \tag{3.18}$$

where V is the wind velocity in km/hr and h_o is the outside film coefficient in kcal/hr-m^2-°C.

In the present work, the value of h_o is obtained for the design day by taking V as the average wind velocity over the day and the external surface as smooth.

Inside film coefficient: The inside film coefficient depends on convection and radiation. The convective component of film coefficient, h_{c_i}, is a function of the temperature drop across the film Δt and is given by the following equations:

(i) For walls in winter or summer

$$h_{c_i} = (0.274 \times 4.88)\ (\Delta t)^{0.25} \tag{3.19}$$

(ii) For floor in winter and ceiling in summer

$$h_{c_i} = (0.126 \times 4.88)\,(\Delta t)^{0.25} \qquad (3.20)$$

(iii) For ceiling in winter and floor in summer

$$h_{c_i} = (0.295 \times 4.88)\,(\Delta t)^{0.25} \qquad (3.21)$$

where h_{c_i} = inside convective film coefficient in kcal/hr-m^2-°C

and Δt = temperature drop across the film, °C.

The inside film coefficient corresponding to radiation component, h_{r_i}, is given by

$$h_{r_i} = (4.88 \times 0.1714)\,\frac{e_1 e_2}{\Delta t}\left[\left(\frac{T_{is} + \Delta t}{100}\right)^4 - \left(\frac{T_{is}}{100}\right)^4\right] \qquad (3.22)$$

where e_1 = emissivity of the transmitting surface

e_2 = emissivity of the surrounding surface

Δt = temperature drop across the film in °C and

T_{is} = absolute temperature of inside surface in °K.

The film coefficients h_{c_i} and h_{r_i} cannot be calculated from Eqs. (3.19) to (3.22) as they are functions of the temperature drop Δt across the film, while Δt itself is dependent on h_{c_i} and h_{r_i}. Hence a trial and error procedure has to be adopted using a standard value of the combined (radiation plus convection) film coefficient and then computing Δt. This approximate Δt is then used in Eqs. (3.19) to (3.22) to obtain the corresponding values of h_{c_i} and h_{r_i}. The later values are then used to obtain a second approximation to Δt, which would give more

accurate values of h_{c_i} and h_{r_i}. This process is continued till the values of h_{c_i} and h_{r_i} in two successive iterations do not differ by more than 0.5%. Then the total inside film coefficient h_i will be equal to the sum of h_{c_i} and h_{r_i}. However, the inside film coefficient in the present work is assumed to be a constant since it is not possible to evaluate the temperature drop across the film, Δt, using Threlkeld method [1] for heat gain.

3.4 Heat Gain Through Sources Within the Conditioned Space

The various factors contributing to the heat gain inside the conditioned space are air changes, people, lights, equipment products. These are discussed below.

3.4.1 Air changes

Each time the door is open, some outside air enters the storage room. The temperature of this warm air must be reduced to the storage room temperature which adds to the heat gain. The traffic in a refrigerated room usually varies with its size or volume. The number of times the doors are opened is dependent upon the volume rather than the number of doors. The values of average air changes per twenty four hours for storage rooms due to door openings and infiltration are reproduced in Table B.3 of Appendix B from reference [15]:

$$\text{The infiltration load} = \frac{p \times V \times (q_o - q_{in})}{R \times T_i} \qquad (3.23)$$

where p = pressure of inside air

V = volume of the air = number of changes per twenty four hours x volume of the ware-house

q_o = heat associated with outside air at an absolute temperature of T_o

q_i = heat associated with inside air at the absolute temperature T_i and

R = universal gas constant.

3.4.2 People

The rate at which heat and moisture are given off by human beings under different states of activity is given in reference [14,15]. In many applications these sensible and latent heat gains constitute a large fraction of the total load. Appreciable variations in the heat emission rates must be recognized according to the age and sex of the individual, state of activity, environmental influences and the duration of occupation. In a warehouse the people are assumed to do moderately heavy work during loading and unloading. The corresponding value of the heat gain is taken from Table B.4 of Appendix B.

3.4.3 Lights

The instantaneous rate of heat gain from electrical lighting, q_{el}, is computed from the following equation [14,15,16].

$$q_{el} = \text{Total light wattage x use factor x special allowance factor x } \frac{3.41}{(2.2046 \times 1.8)} \text{ kcal/hr} \qquad (3.24)$$

The total light wattage is obtained from the ratings of all fixtures installed. The use factor in Eq. (3.24) is the ratio of wattage in use, for the conditions under which the load estimate is being made, to the installed wattage. The special allowance factor is introduced to take care of fluorescent fixtures and for fixtures which are either ventilated or installed so that only part of their heat goes to the conditioned space. For fluorescent fixtures, the special allowance factor is taken as 1.20 in order to allow for the power consumed in the ballast. For ventilated fixtures, recessed fixtures and the like, manufacturer's data must be sought to establish the fraction of the total wattage which is expected to enter the cooled space.

3.4.4 Equipments[15]

When equipment of any sort is operated within the conditioned space by electric motors, the heat equivalent of this operation must be considered in the calculation of heat gain. The equation for calculating this heat gain is given by

$$q_{em} = \frac{\text{Horse power rating x load factor x 2545}}{\text{Motor efficiency x (2.2046 x 1.8)}} \qquad (3.25)$$

where it is assumed that both the motor and the driver equipment are within the conditioned space. If the motor is outside the space than the factor "motor efficiency" will not appear in Eq. (3.25). The load factor is merely the fraction of the rated load which is delivered under the conditions in which the cooling load estimate is made. The motor efficiency may be approximated as 50 to 60 percent at 1/8 horse-power rating, increasing to 80 percent at 1.0 horse-power and 88 percent at 10 horse-power and above as given in reference [15].

3.4.5 Products

A product placed in a refrigerated warehouse at a temperature higher than the storage temperature will lose heat until it reaches the storage temperature. The quantity of heat removed may be calculated from a knowledge of the product, including the state while entering the cold room, final state, weight, specific heat above and below freezing, freezing temperature and latent heat of the product. When the product is cooled from one state and temperature to another state and temperature, the following equations can be used for calculating the heat gain.

Heat removed, q_i, for cooling from an initial temperature t_1 to some lower temperature t_2 above freezing:

$$q_i = m \times C \times (t_1 - t_2) \tag{3.26}$$

Heat removed, q_f, from an initial temperature t_1 to the freezing point, t_f, of the product:

$$q_f = m \times C \times (t_1 - t_f) \tag{3.27}$$

Heat removed, q_{if}, while freezing the product:

$$q_{if} = mh_{if} \tag{3.28}$$

Heat removed, q_{ff}, from the freezing point to the final temperature t_3 below freezing:

$$q_{ff} = m \times C_i \times (t_f - t_3) \tag{3.29}$$

where m is the mass of the product, C and C_i are the specific heats of the product above and below the freezing point, t_f is the freezing temperature and h_{if} is the latent heat of freezing.

3.4.6 Heat of respiration

All fruits and vegetables are alive; they continue to undergo changes during storage. The most important of these changes is produced by respiration. During this process energy is released in the form of heat. The amount of heat liberated varies with the type and temperature of the product. The colder is the product, the lesser will be the heat of respiration. The rate of evolution of heat of the product is given in Table B.5 of Appendix B taken from reference [15].

3.5 Rate of Loading and Unloading

It is important to consider the rate of loading and unloading the warehouse in computing the heat gain. It is assumed that the warehouse will be loaded to its full capacity in one month, with twelve working hours per day, as shown in Figure 3.2(a).* Then the average mass of the product loaded per hour will be

$$\dot{m}_h = \frac{1}{2} \times \text{number of hours of loading} \times \frac{\text{average rate of loading (mass)}}{\text{number of hours per day}}$$

$$= \frac{1}{2} \times \frac{12 \times \dot{m}}{24} = \frac{\dot{m}}{4} \tag{3.30}$$

where $\dot{m}_h$ is the average mass of the commodity per hour and $\dot{m}$ is the actual rate of loading (mass per hour).

It is assumed that loading is done only for twelve hours a day. Thus the sensible heat load is given by

$$q_s = \dot{m}_h \times C \times (t_1 - t_2) \tag{3.31}$$

This sensible heat load will be only for the period of loading. It is assumed that in the next twelve hour of the day the temperature of the commodity will come down to the storage temperature.

*This model of loading is assumed as most of the refrigerated warehouses in India are loaded once to their full capacity with the seasonal commodity (like potato) and then the commodity is preserved for a long time. However other models like continuous loading and unloading can also be assumed if needed.

Respiration heat gain: For the respiration energy, the weight of the product $\dot{m}_r$ is computed as

$$\dot{m}_r = \left(\frac{\dot{m}\tau}{2} + \dot{m}\tau\right) + \left(\dot{m}\, 2\tau + \frac{\dot{m}\tau}{2} + \dot{m}\tau\right) + \ldots + [4(n-1) + 3]\,\frac{\dot{m}\tau}{2}$$

$$= \frac{\dot{m}\tau}{2}\,[3 + 7 + \ldots + 4(n-1) + 3\,]$$

$$= \frac{\dot{m}\tau}{2} \cdot \frac{n}{2}\,[6 + (n-1)4\,] = \frac{\dot{m}\tau}{2} \cdot \frac{n}{2}(4n + 2) \qquad (3.32)$$

where n is the number of days of loading and τ is the number of hours of loading per day, taken as equal to 12 in this case.

Thus the average mass of the product per hour will be

$$\dot{m}_r = \frac{\frac{\dot{m}x12}{2} \cdot \frac{n}{2}(4n + 2)}{n \times 24} = \frac{\dot{m}}{4}(2n + 1) \qquad (3.33)$$

The mass per hour $\dot{m}_r$ is assumed to be valid only during loading period. Once the warehouse is loaded to its full capacity, then the mass per hour will be equal to the total mass of the commodity stored. When all the above-mentioned heat gains are added, one gets the total heat gain. This heat gain is to be converted to the corresponding cooling load as outlined below.

3.6 Cooling Load

The hourly heat gain is to be converted into the corresponding cooling load as the radiations absorbed by the interior furnishings and the structure reach the conditioning equipment after a considerable delay in time. There are three

methods available for this purpose: (i) Exact method [55], (ii) Approximate method known as weighting factor method [15,52] and (iii) Another approximate method [5] known as the transfer function method.

3.6.1 Exact method

This method was given by Kududa and Powell [55] to convert the heat gain into cooling load. It requires a laborious solution of energy balance equations involving room air, surrounding walls, infiltrating and ventilation air and internal energy sources. The principle of calculation is based on considering a fictitious space that is enclosed by four windowless walls, a ceiling and floor and having no infiltration or ventilation air and internal energy sources. The six equations that govern energy exchange at each of the inside surfaces at any time are given by

$$q_{i,\tau} = h_{c_i}(t_{i,\tau} - t_{a,\tau}) + \sum_{i \neq j}^{6} g_{ij}(t_{i,\tau} - t_{j,\tau}) \qquad \text{for } j = 1,\ldots,6 \qquad (3.34)$$

where $q_{i,\tau}$ = rate of heat conducted out of surface i at the inside surface at time τ

h_{c_i} = convective heat transfer coefficient at the interior surface i

g_{ij} = radiation heat transfer factor between the interior surfaces i and j

$t_{a,\tau}$ = inside air temperature at time τ

$t_{i,\tau}$ = uniform temperature of the interior surface i at time τ

$t_{j,\tau}$ = uniform temperature of the interior surface j at time τ

The equations governing conduction within the six slabs cannot be solved independently of the above equations since the energy exchanges occurring within the room affect the inside surface conditions which in turn affect the internal conditions. Consequently one is faced with the problem of solving six equations simultaneously along with the governing equations of conduction in six slabs in order to calculate the cooling load at any time τ ($Q_{L,\tau}$). $Q_{L,\tau}$ can be expressed as

$$Q_{L,\tau} = \sum_{i=1}^{6} h_{c_i}(t_{i,\tau} - t_{a,\tau}) \tag{3.35}$$

A rigorous approach such as this for calculating the cooling load would be practically impossible, especially when it has to be incorporated in an automated optimum design programme.

3.6.2 Weighting factor method

In order to simplify the calculation procedure for obtaining the cooling load, various approximate methods have been devised to take the instantaneous heat gain components, convert them to load components and add these components together to obtain the instantaneous load. Generally these

methods assume a certain fraction of the heat gain to be convective and/or latent (and consequently felt by the space air instantaneously) and the remaining fraction to be radiative which would not be felt until sometime later (after it has been absorbed by the interior surfaces and subsequently convected to the air). The general name associated with these methods is "weighting factor methods". The simplest one is given in Table C.1 of Appendix C, wherein all the components of heat gain that normally occur in a space are listed along with the percentage of that gain which can be considered radiant, convective and latent. The cooling load component corresponding to the heat gain is then given by the following equation

$$\begin{bmatrix}\text{Total value of heat gain}\\ \text{component at the time}\\ \text{under consideration}\end{bmatrix} \times \begin{bmatrix}\text{\% component of gain which}\\ \text{is convective plus latent}\\ \text{(expressed as a fraction)}\end{bmatrix} +$$

$$\begin{bmatrix}\text{Average value of heat}\\ \text{gain component over 3, 5}\\ \text{or 7 hour period upto and}\\ \text{including the time of}\\ \text{calculation}\end{bmatrix} \times \begin{bmatrix}\text{\% of component gain}\\ \text{which is radiant}\\ \text{(expressed as a}\\ \text{fraction)}\end{bmatrix} \qquad (3.36)$$

3.6.3 Transfer function method

Mitalas [5] has given the transfer function method for obtaining cooling load from heat gain. This method is nothing more than a modified weighting factor method for obtaining the cooling load from heat gain. A completely different set of factors (or coefficients of room transfer function) are given for different types of structures (like light, medium and

heavy structures) and in addition, different weights are given to different hourly heat gain values included in the averaging scheme. The heat gain $q_{e,\tau}$ is given in the form of a time series, which indicates the values of heat gain at equally spaced points in time. The corresponding cooling load Q_τ at any time τ can be related to the current value of heat gain $q_{e,\tau}$ and the preceeding values of cooling load and heat gain as

$$Q_\tau = \sum_{i=1} (v_0 q_{e,\tau} + v_1 q_{e,\tau-\Delta} + v_2 q_{e,\tau-2\Delta} + \ldots) - (w_1 Q_{\tau-\Delta} + w_2 Q_{\tau-2\Delta} + w_3 Q_{\tau-3\Delta} \ldots) \tag{3.37}$$

where i denotes the number of heat gain component and Δ represents the time interval. The quantities $v_0, v_1, \ldots, w_1, w_2, \ldots$ indicate the coefficient of transfer function given in Table C.2 of Appendix C.

In the present work the hourly heat gain components are converted to the corresponding cooling load components using the transfer function method.

3.6.4 Cooling load computation

The cooling load is calculated for all the twenty four hours of the critical day for each month. The load during an hour is taken as the arithmetic mean of the six values descending from the maximum value. The actual load is taken as the weighted average of the twelve hourly load values calculated,

one corresponding to each month of the year. The weights for different months are computed as follows:
If t_{e_i} denotes the maximum average monthly solar-air temperature and t_i the inside temperature, the weight for the month i, W_i, is computed as

$$W_i = (t_{e_i} - t_i) \tag{3.38}$$

These weights are normalized as

$$W_{in} = \frac{W_i}{W} \qquad \text{for } i = 1,\ldots,12 \tag{3.39}$$

where t_{e_i} = maximum average solar-air temperature in the month i

t_i = inside dry bulb temperature

W_{in} = normalized weight corresponding to the month i

$W = \sum_{i=1}^{12} W_i$.

Thus the total hourly cooling load, Q, valid over the year is computed as

$$Q = \sum_{i=1}^{12} W_{in} Q_i \tag{3.40}$$

where Q_i is the hourly cooling load for the month i.

3.7 Economics Models [40]

The following four economics models are generally considered in the analysis of any investment policy. Any of

them can be used to indicate the most favourable choice amongst several alternatives. Some of their merits and demerits are discussed briefly in the following paragraphs.

3.7.1 Present-worth method

In this method all the costs and incomes are translated into the present worth. In order to determine the present worth of a future sum, first the interest rate must be established. Then present worth is the value of a sum of money at the present time that, with compounded interest, will have a specified value at a certain time in future. Thus

$$P = \frac{S}{(1 + r)^n} = (PWF)S \qquad (3.41)$$

where P = principal

r = rate of interest per period

S = total amount to be repayed at future time

n = number of periods and

PWF = single-payment present worth factor.

The series present worth is given by

$$P = R\left[\frac{(1 + r)^n - 1}{r(1 + r)^n}\right] = (R)(SPWF) \qquad (3.42)$$

where P = original amount

R = amount withdrawn at the end of first and subsequent periods and

SPWF = series present worth factor.

Although the present-worth method is widely used, it presents some difficulty if lives of two possible investments are different.

3.7.2 Annual cost method

It translates all non annual costs to an annual cost basis. In this method also the rate of interest must be established first.

$$R = P\left[\frac{r(1+r)^n}{(1+r)^n-1}\right] = P(CRF) \tag{3.43}$$

where CRF = the annual recovery factor.

The annual cost method has two advantages over the present worth method. One is that there is no complication introduced when the prospective investments have different lives. The second is that it is more natural for most people to think in terms of an annual cost than in terms of the present worth.

3.7.3 Rate of return method

In this method no assumption is made regarding the interest rate. Instead the money for the investment is considered to be in hand, and the rate of return is calculated as though it were an interest rate received by making an outside investment.

Here we find r such that

$$P(CRF) - \text{Salvage value } (SFF) \quad = \quad \text{Net income} \tag{3.44}$$

where SFF = sinking fund factor = $\left[\dfrac{r}{(1+r)^n-1}\right]$.

3.7.4 Break-even point

This method assumes that money is borrowed at a specific rate of interest and that the loan is paid off as rapidly as possible with no profits extracted. The break-even point is defined as the time where the loan is paid off and profits begin following to the investor. In contrast to the rate of return method where the interest rate was unknown, the life is unknown in this method. Thus one finds n such that

$$P(CRF) - \text{Salvage value } (SFF) \quad = \quad \text{Net income} \tag{3.45}$$

3.7.5 Annual cost with simple interest

A simple economics model of annual cost with simple interest instead of compound interest is also considered in few cases. Here

$$R \quad = \quad \frac{P(1 + n*r)}{n} \tag{3.46}$$

In the present work, the present worth and the annual cost with compound interest and simple interest are considered to decide about the investment policy for some simple one variable and two variable optimization problems. However, only the annual cost method with compound interest is used in further analysis.

3.8 Thermal Insulation [15,56]

The requirement of using thermal insulation in refrigerated warehouses or air-conditioned buildings is a result of simple economics principle, that is, to minimize the cost of equipment necessary to provide the desired cooling load. In addition, thermal insulation is required to prevent surface condensation. The choice of an insulating material usually involves a compromise with regard to several desirable properties like low thermal conductivity, low heat capacity and, sometimes, strength. Consideration should also be given to the properties of insulation related to the health and safety including its inability to support vermin and insects, freedom from fire hazards, dust or objectionable vapours, loose particles that may be irritating to the skin, odour, resistance to decay and resistance to odour absorption and retention. Moisture should be kept away from insulation, to avoid deleterious effects.

The factors like ease of installation, although, not a physical property, is so important that it cannot be over stressed. Generally the designer should select those thermal insulating materials which can be properly installed with a minimum of skill. The expanded polystyrene and mineral wool are generally used for low temperature and long life applications.

3.9 Water Vapour Barriers [15]

There are materials which provide resistance to the transmission of water vapour under specified conditions. They are classified as structural membrane or coating barriers. A water vapour barrier does not necessarily stop the flow of vapour, but serves as a medium of control to reduce the rate and volume of flow. These water vapour barriers prevent the accumulation of water within the insulation or construction. By retarding the transmission of water vapour, water vapour barriers help to (a) keep the insulation dry, and reduce the heat load requirements for the cooling system, (b) prevent structural damage by rot, corrosion or expansion effect of freezing water, and (c) reduce paint problems.

The primary source of moisture is outdoor air which enters while opening and closing the doors, cracks and joints in the outdoor structure and discontinuities in the vapour barrier or insulation systems. Many construction materials like mortar, wood and brick are also permeable to air, which can pass through walls when high velocity which impinges on exterior surfaces causing partial vacuums in other portions. In fact the outer surface of any structure is never air tight.

For insulated construction in which the vapour flow may be in either direction for an extended period of time, the insulation must be protected by sealing inside with low permeance barrier. The barrier should be provided with no openings and lap joints as they reduce its effectiveness.

The transmission of water vapour through materials under the influence of a water vapour pressure gradient is called "vapour diffusion". The design equation commonly used for calculating the quantity of water vapour transmitted is given by [15]

$$W = \mu A \tau \frac{\Delta p}{l} \tag{3.47}$$

where W = total weight of vapour transmitted,

μ = permeability,

A = area of cross section of the flow path,

τ = time during which the transmission occurred,

Δp = difference of vapour pressure between the ends of the flow path, and

l = length of the flow path.

Whenever it is convenient to deal with material thicknesses other than unit thickness to which μ refers in Eq. (3.47), use is made of permeance coefficient M, where $M = \frac{\mu}{l}$. The corresponding flow equation is given by

$$W = M A \tau \, \Delta p \tag{3.48}$$

The resistance to water vapour flow through a material is the reciprocal of its permeance and is equal to the thickness divided by its permeability. The overall resistance of an insulated section to the flow of water vapour is the sum of resistances of the various materials in series comprising the section. The forces other than the pressure of water vapour, such as hydraulic pressure, absorption, adsorption,

hygroscopicity and capillarity, affect the water vapour flow and cause moisture migration. These various forces are interrelated but the actual relationship is very complex and has not been expressed in a simple way for use in designs.

In many cases, the consideration of water vapour pressure difference alone can be used to obtain a fairly accurate calculation of water vapour flow. This consideration is useful in designing vapour barrier systems or in preventing the possibility of condensation within a given insulated system.

3.10 Problem Formulation

The problem of optimum design of refrigerated warehouses can be stated in the form of a standard mathematical programming problem as:

$$\text{Find the design vector } \vec{X} = \begin{Bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{Bmatrix} \tag{3.49}$$

which minimizes the merit function $f(\vec{X})$

subjected to the constraints $g_j(\vec{X}) \leq 0$, $j = 1, 2, \ldots, m$

3.10.1 Objective function

The nature of a thermal system design problem is such that there will usually be many designs that perform the

specified functional purposes adequately. The objective or merit function in a thermal design optimization problem represents a basis for choice between alternate acceptable designs. The minimization of cooling and/or heating load and the minimization of initial, plus operating and maintenance costs are taken as the objectives in the present work. In some cases, an objective function which is a linear combination of load (cooling and/or heating) and cost (initial plus operating and maintenance) may be considered for minimization. Whenever the cost is treated as an objective function, the economics model used in the estimation of costs plays an important role. In this work, an attempt is made to find the influence of the economics model on the optimum design of refrigerated warehouses.

3.10.2 Design requirements

The design restrictions are the upper and lower bounds on the insulation thickness and the bounds on the building envelope.

3.10.3 Design variables

The objective and the design requirements being known, the problem of optimum design of a refrigerated warehouse or an air-conditioned building can be cast as a mathematical programming problem once the design variables are chosen. The thicknesses of walls, roof and insulation and the length, width and height of the warehouse are treated as design variables in this work.

3.11 Design Examples

A computer programme has been developed for the automated optimum design of refrigerated warehouses by including all the factors stated in sections 3.1 to 3.8. The programme is quite general and can be used to optimize any refrigerated warehouse having any number of apartments.

The optimum design of the refrigerated warehouse shown in Figure 3.1(a) is considered to illustrate the effectiveness of the optimization procedure developed. The warehouse (with 8 apartments inside) is assumed to have a known volume with all the four walls and roof exposed to atmosphere. The walls and roof are supposed to be constructed of bricks and reinforced concrete respectively with a 12 mm plaster on each side. All the walls and roof are insulated from inside. Two type of commonly used insulation, namely, expanded polystyrene and mineral wool are considered. The thermal properties of different materials and other design data are given in Table 3.2. The various aspects pertaining to the deterministic design of refrigerated warehouses are studied in six groups. The first four groups are devoted to parametric study. In the first group of problems, the thicknesses of roof and insulation are taken as design variables. Two types of insulation materials, as discussed earlier, are studied from economical point of view. The economical insulation material so found is used in subsequent design problems. In group two, three brick thicknesses are

considered for the insulated walls. The strength of walls or roof is not considered; the design is based only on thermal considerations. In the third group, the absorptance of the exposed surface is considered as a design variable although it could not be related to the cost in the absence of availability of cost data. After selecting the optimal thicknesses of walls and roof and the type of insulation, the warehouse orientation was changed (three different positions are chosen) for the same location in group four. Once the economical orientation is chosen, the optimal warehouse envelope is determined for a given volume in group five. The dimensions of the warehouse are taken from an existing one at Kanpur, India. Finally in group six, a realistic problem is solved with all the internal loads. In all the above problems, the merit function is taken as the total cost, i.e., the sum of initial, running and maintenance costs. Another problem in group six is solved by considering the total cooling load as the objective for minimization. The statement and results of various optimization problems are given below.

3.11.1 Group I

The determination of optimum thicknesses of R.C.C. slab and its insulation is considered. The problem is solved for two different types of insulation. Two economics models, namely the annual cost with compound interest and the annual cost with simple interest, are used for each problem. Each of the problems can be stated as follows:

$$\text{Find } \vec{X} = \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix}$$

such that $f(\vec{X}) = f_1(\vec{X}) + f_2(\vec{X}) + f_3(\vec{X}) + f_4(\vec{X})$ minimum

subjected to
$$g_1(\vec{X}) = X_1 - 0.2 \leq 0$$
$$g_2(\vec{X}) = 0.10 - X_1 \leq 0$$
$$g_3(\vec{X}) = X_2 - 0.2 \leq 0$$
$$g_4(\vec{X}) = 0.005 - X_2 \leq 0$$

where X_1 = R.C.C. slab thickness, X_2 = insulation thickness for roof (flat), $f_1(\vec{X})$ = equipment cost, $f_2(\vec{X})$ = insulation cost, $f_3(\vec{X})$ = operating and maintenance cost, and $f_4(\vec{X})$ = cost of R.C.C. slab. The costs of R.C.C., insulation and equipment are given in Table 3.2. These costs are converted to annual costs using Eqs. (3.43) and (3.46) for compound and simple interests respectively. The details of the four problems solved are given in Table 3.3(a) and the results of optimization in Table 3.3(b).

It can be seen that in all the cases, the lower bound on concrete thickness is active at the optimum point. The two economics models give different results indicating their inherent characteristics. Since the optimum total cost with mineral wool as insulating material is lower than that with expanded polystyrene, mineral wool is recommended for insulation. The progress of optimization procedure is shown in Figure 3.3 for problem 1.

3.11.2 Group II

In this group three problems, each with a different brick thickness, are solved with mineral wool as the insulating material. Two different economics modes, namely the annual cost and the present worth, are considered for each problem. The optimization problems are formulated with insulation thickness as design variable as follows:

$$\text{Find } \vec{X} = \{X_1\}$$

$$\text{such that } f(\vec{X}) = f_1(\vec{X}) + f_2(\vec{X}) + f_3(\vec{X}) + f_4 \rightarrow \text{minimum}$$

$$\text{subjected to } g_1(\vec{X}) = X_1 - 0.2 \leq 0$$

$$g_2(\vec{X}) = 0.005 - X_1 \leq 0$$

where X_1 = insulation thickness, $f_1(\vec{X})$ = equipment cost, $f_2(\vec{X})$ = insulation cost, $f_3(\vec{X})$ = operating and maintenance cost and f_4 = cost of bricks. The costs are given in Table 3.2. These costs are converted to annual cost and present-worth using Eqs. (3.43) and (3.42) respectively. The characteristics of the optimization problems are summarized in Table 3.4(a) and the results of optimization are shown in Table 3.4(b).

Although the cost and energy requirements are minimum with 22.5 cm brick wall, due to the large envelope of the warehouse, the brick thickness of 33.75 cm is recommended for practical wall construction. This brick thickness corresponds to an increase of 4% in annual cost and 5% in energy requirement

while the 45.0 cm brick thickness makes a 7% increase in annual cost and hardly 1% increase in energy requirement. Thus, if minimization of energy is taken as design criterion, then probably 45.0 cm brick thickness might be more suitable than 33.75 cm thick brick wall. The progress of optimization procedure for problem 9 is shown in Figure 3.4.

3.11.3 Group III

A problem with insulation thickness and absorptance of exterior surface of the wall as design variables is solved in this group to assess the effect of absorptance of the surface. The brick thickness of the wall is taken as 33.75 cm and the insulation material is assumed as mineral wool. The annual cost is taken as the economics model. This problem is formulated as follows:

$$\text{Find } \vec{X} = \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix}$$

$$\text{such that } f(\vec{X}) = f_1(\vec{X}) + f_2(\vec{X}) + f_3(\vec{X}) \rightarrow \text{minimum}$$

$$\text{subjected to } g_1(\vec{X}) = X_1 - 0.2 \leq 0$$

$$g_2(\vec{X}) = 0.005 - X_1 \leq 0$$

$$g_3(\vec{X}) = X_2 - 0.85 \leq 0$$

$$g_4(\vec{X}) = 0.10 - X_2 \leq 0$$

where X_1 = insulation thickness, X_2 = value of absorptance, $f_1(\vec{X})$ = equipment cost, $f_2(\vec{X})$ = insulation cost, and $f_3(\vec{X})$ = operating and maintenance cost. These costs are converted to the annual cost by using Eq. (3.43). It is to be noted that the cost associated with absorptance (X_2) is not included in the above objective function.

Since the lower bound on absorptance became active at the optimal point, it appears that lower values of absorptance would be more economical. However, the cost of achieving and maintaining any specified value of absorptance cannot be estimated with the available data. The progress of optimization procedure is shown in Figure 3.5.

3.11.4 Group IV

In this group the effect of orientation of the building is studied. The warehouse is assumed to be in three different orientations at the same location as shown in Figure 3.2(b) and the optimal cost corresponding to each orientation (to maintain the desired inside conditions) is obtained. Mineral wool is considered as the insulating material and the annual cost economics model is used. The optimization problem is formulated as:

$$\text{Find } \vec{X} = \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{Bmatrix}$$

such that $f(\vec{X}) = f_1(\vec{X}) + f_2(\vec{X}) + f_3(\vec{X}) \rightarrow$ minimum

subjected to

$$0.005 \leq X_1 \leq 0.2$$

$$0.005 \leq X_2 \leq 0.2$$

$$0.005 \leq X_3 \leq 0.2$$

$$0.005 \leq X_4 \leq 0.2$$

$$0.005 \leq X_5 \leq 0.2$$

where X_1 = insulation thickness for roof, $X_2, \ldots, X_5$ = insulation thicknesses for the four walls, $f_1(\vec{X})$ = equipment cost, $f_2(\vec{X})$ = insulation cost and $f_3(\vec{X})$ = operating plus maintenance cost. The result of optimization are shown in Table 3.6.

By using the optimal roof and wall thicknesses obtained in earlier problems, the effect of orientation of the warehouse at a particular location is considered. The three orientations considered are shown in Figure 3.2(b). It is found from optimal design that the total cost is minimum for orientation 3, while the energy requirements are minimum for orientation 2. Though the cost reduction is hardly 0.6% for orientation 3 compared to orientation 2, the energy requirement is reduced by 1.6% for orientation 2 compared to that of orientation 3. Thus orientation 2 is preferred for the warehouse at the given location. The progress of optimization path for problem 12 is shown in Figure 3.6.

3.11.5 Group V

The determination of optimum envelope of the warehouse, i.e., finding optimal length, width and height of the warehouse, for a given volume and insulation thicknesses for roof and walls is considered in this group with the minimization of total cost as the objective. Only the cooling load due to building structure is considered. The annual cost model is used for the investment policy and mineral wool is considered as insulation material. The mathematical formulation of the problem is as follows:

$$\text{Find } \vec{X} = \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \\ X_7 \end{Bmatrix}$$

$$\text{such that } f(\vec{X}) = f_1(\vec{X}) + f_2(\vec{X}) + f_3(\vec{X}) \rightarrow \text{minimum}$$

subjected to

$$0.005 \leq X_1 \leq 0.2$$

$$0.005 \leq X_2 \leq 0.2$$

$$0.005 \leq X_3 \leq 0.2$$

$$0.005 \leq X_4 \leq 0.2$$

$$0.005 \leq X_5 \leq 0.2$$

$$0.6 \leq X_6 \leq 1.2$$

$$0.3 \leq X_7 \leq 0.6$$

$$(0.455/X_6 * X_7) - 0.15 \leq 0$$

$$0.1 - (0.455/X_6 * X_7) \leq 0$$

where X_1 = insulation thickness for roof, $X_2, \ldots, X_5$ = insulation thicknesses for four walls, X_6 = length of the warehouse, X_7 = width of the warehouse, $f_1(\vec{X})$ = equipment cost, $f_2(\vec{X})$ = insulation cost and $f_3(\vec{X})$ = operating and maintenance cost. The design variables X_6 and X_7 are normalized and the volume of the warehouse is assumed to be 455,000 m^3. The results of optimization are given in Table 3.7.

It is found that, for a given volume of the warehouse, it is economical to have the maximum permissible height. The aspect ratio at the optimum point has been found to be approximately 1.5. The energy requirements are reduced with the optimal envelope dimensions and insulation thicknesses by 10% while the aspect ratio changed from approximately 2.0 to 1.5 and the height from 13.0 m to 14.9 m. The progress in the optimization procedure is shown in Figure 3.7.

3.11.6 Group VI

In this group a realistic problem with all internal and external cooling loads is solved with two different merit functions. In the first case, the building envelope parameters and insulation thicknesses for walls and roof are optimized for

minimum total cost while in the second case, the same problem is solved for minimum total cooling load. The annual cost is taken as the economics model for the investment policy. The insulating material is assumed to be mineral wool. The mathematical formulation of the problem 17 is as follows:

$$\text{Find } \vec{X} = \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \\ X_7 \end{Bmatrix}$$

such that $f(\vec{X}) = L_c \rightarrow$ minimum, subjected to

g_1, g_2 : $0.005 \leq X_1 \leq 0.15$

g_3, g_4 : $0.005 \leq X_2 \leq 0.15$

g_5, g_6 : $0.005 \leq X_3 \leq 0.15$

g_7, g_8 : $0.005 \leq X_4 \leq 0.15$

g_9, g_{10}: $0.005 \leq X_5 \leq 0.15$

g_{11}, g_{12}: $0.6 \leq X_6 \leq 1.2$

g_{13}, g_{14}: $0.3 \leq X_7 \leq 0.6$

g_{15} : $(0.455/X_6 * X_7) - 0.15 \leq 0$

g_{16} : $0.1 - (0.455/X_6 * X_7) \leq 0$

where L_c is the total cooling load and all other symbols have the same meaning as in Group V. The results of optimization are given in Table 3.8.

In problem 16, the constraint corresponding to the upper bound on height became active while in problem 17, the upper bounds on the thickness of roof and two walls became active. The optimum point in problem 17 corresponds to an increase of 5.15% in cost while the cooling load is reduced by 7.30%. In this case, the aspect ratio changed from 1.50 to 1.75 while the height reduced from 14.9 m to 13.88 m. The progress in optimization procedure for the problem 16 is shown in Figure 3.8.

3.12 Kuhn-Tucker Conditions

As stated in Chapter 2, for a constrained minimum, the Kuhn-Tucker conditions given by Eqs. (2.17) and (2.18) should be satisfied. Let

$$\vec{E} = -\nabla f(\vec{X}^*)$$

$$[D] = [D_{ij}] = \left(\frac{\partial g_j}{\partial X_i}\right)$$

where $i = 1, \ldots, n$,

$j \in J$, and

J is the set of indices of active constraints

then the value of $\vec{\xi}$ can be obtained as

$$\vec{\xi} = ([D]^T [D])^{-1} [D]^T \vec{E} \tag{3.50}$$

To illustrate that the final design points $\vec{X}^*$ obtained in different problems are optimum, Kuhn-Tucker conditions are applied to problem 17 (group VI) in which constraints 1, 5 and 9 are active. For this problem $\vec{E}$ and $[D]$ are found to be

$$\vec{E} = \begin{Bmatrix} 14.35 \\ 7.82 \\ 12.80 \\ 8.44 \\ 33.46 \\ 4.42 \\ 3.70 \end{Bmatrix}$$

and $$[D] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The vector $\vec{\xi}$ given by Eq. (3.50) is computed as

$$\vec{\xi} = \begin{Bmatrix} \xi_1 \\ \xi_5 \\ \xi_9 \end{Bmatrix} = \begin{Bmatrix} 14.35 \\ 12.80 \\ 33.46 \end{Bmatrix} > \vec{0}$$

Since all ξ_i are positive, Kuhn-Tucker conditions are satisfied and hence $\vec{X}^*$ is guaranteed to be a relative minimum.

3.13 Sensitivity Analysis

In practice, a designer would be interested in knowing how the total cost or cooling load varies with a change in the design parameters. This type of sensitivity analysis will help the designer in manipulating the desin variables to suit some specific requirements. Further in some cases, the results obtained from the optimization procedure may have to be rounded-off to the nearest practical values of the design variables. Hence a sensitivity analysis of the cooling load/total cost with respect to different design variables is conducted at the optimum point. Thus the reference design is taken as the optimal design, and the values of the design parameters are changed by 50% in steps of 10% on the positive and negative sides. The results of the sensitivity analysis for problem 17 are shown in Figures 3.9 and 3.10. It can be seen from these figures that the cooling load is most sensitive to changes in the insulation thickness of the roof (X_1) as well as the width of the building (X_7) and least to changes in thickness of insulation of one of the walls (X_2). The total cost is most sensitive with respect to the length of the warehouse (X_6) and least with respect to X_2.

TABLE 3.1

SELECTION OF DESIGN DAY FOR NEW DELHI 1967

Month	Date	Dry bulb temperature		Solar radiation		Solar-air temperature	
		Mean value °C	Standard deviation	Mean value $kcal/hr\text{-}m^2$	Standard deviation	Mean value °C	Standard deviation
January	24	15.94	6.70	217.18	214.05	23.18	14.65
February	28	21.95	6.47	288.81	264.14	31.58	16.53
March	1	24.33	5.20	298.87	269.82	34.29	16.49
April	17	31.40	4.50	376.00	301.12	43.93	17.66
May	31	35.90	5.09	420.87	309.42	49.92	18.92
June	5	36.96	3.05	406.62	300.22	50.51	16.90
July	14	33.20	2.68	378.50	283.15	45.82	15.62
August	1	30.94	2.47	370.69	290.41	43.29	15.64
September	20	29.53	2.72	348.81	288.30	41.15	15.57
October	1	28.45	3.24	322.56	275.19	39.21	15.50
November	1	22.10	4.19	260.62	246.98	30.80	14.51
December	14	17.67	2.47	190.19	195.02	24.01	10.53

TABLE 3.2

DESIGN DATA

(1)	Insulation	: Life : 10 years Cost of mineral wool: Rs. 440/m thickness/m^2 Cost of polystyrene : Rs. 650/m thickness/m^2 Fixing charges for insulation: Rs. 2/m^2
(2)	Equipment	: Life : 25 years Cost : Rs. 2000/ton of refrigeration Running and maintenance cost: Rs. 2000/3000 kcal/hr of refrigeration/annum
(3)	Bricks	: Life : 50 years Cost : Rs. 150/thousand Dimensions: 0.2250 m X 0.1125 m X 0.0750 m
(4)	R.C.C.	: Life : 100 years Cost : Rs. 350/m thickness/m^2
(5)	Rate of interest	: 10% per annum
(6)	Warehouse	: Volume : 455,000 m^3 Internal temperature : 0°C Relative humidity : 95% Latitude : 29°

Continued...

Table 3.2 (Continued)

		A	E	C	F
(7)	Orientations :				
	1 :	(0,0,1)	(0,-1,0)	(0,0,-1)	(0,1,0)*
	2 :	(0,0.707,0.707)	(0,-0.707,0.707)	(0,-0.707,-0.707)	(0,-0.707,-0.707)
	3 :	(0,1,0)	(0,0,1)	(0,-1,0)	(0,0,-1)

(8) Areas : Walls A and C = 1120 m^2
Walls E and F = 545 m^2
Roof = 3600 m^2

(9) Thermal properties:

Material	Thermal conductivity kcal/hr-m-°C	Diffusivity m^2/hr
Brick	0.66	1.74×10^{-3}
Plaster	0.72	2.25×10^{-3}
R.C.C.	1.10	1.77×10^{-3}
Expanded polystyrene	0.032	2.5×10^{-3}
Mineral wool	0.035	1.6×10^{-3}

*Figures in brackets indicate the direction cosines of normals to the four walls of the warehouse (Figure 3.1(b)).

TABLE 3.3(a)

DETAILS OF PROBLEMS CONSIDERED IN GROUP I

Problem number	Design variables	Economics model	Criterion function	Constraints	Type of insulation
1 and 2	R.C.C. roof thickness Insulation thickness of roof	Annual cost with compound interest for problem 1 Annual cost with simple interest for problem 2	R.C.C. + Insulation + Equipment + Running and Maintenance cost	Upper and lower bounds on R.C.C. and insulation thicknesses	Mineral wool
3 and 4	R.C.C. roof thickness Insulation thickness of roof	Annual cost with compound interest for problem 3 Annual cost with simple interest for problem 4	R.C.C. + Insulation + Equipment + Running and Maintenance cost	Upper and lower bounds on R.C.C. and insulation thickness	Expanded polystyrene

TABLE 3.3(b)

OPTIMIZATION RESULTS FOR PROBLEMS CONSIDERED IN GROUP I

Problem number	At initial design			At optimum design			% reduction in cost	Computer time	Cooling load at optimum design (kcal/hr)
	Design variables (m)	Penalty function	Objective function (Rs.)	Design variables (m)	Penalty function	Objective function (Rs.)			
1	$X_1 = 0.11$ $X_2 = 0.01$	2,29,806.9	1,67,325.7	$X_1=0.1008^*$ $X_2=0.0975$	67,795.4	66,795.0	65.0	34 minutes on IBM 7044 computer	36,625.2
2	$X_1 = 0.11$ $X_2 = 0.01$	2,22,449.5	1,59,968.3	$X_1=0.1034^*$ $X_2=0.0882$	61,995.3	61,930.8	61.4	32 minutes on IBM 7044 Computer	40,084.5
3	$X_1 = 0.11$ $X_2 = 0.01$	2,34,190.7	1,69,709.5	$X_1=0.1008^*$ $X_2=0.0755$	74,974.3	74,723.1	56.2	32 minutes on IBM 7044 computer	42,591.3
4	$X_1 = 0.11$ $X_2 = 0.01$	2,28,531.3	1,66,302.5	$X_1=0.1044^*$ $X_2=0.0684$	70,802.9	70,751.9	57.4	30 minutes on IBM 7044 computer	46,484.1

* Constraint corresponding to lower bound on R.C.C. thickness became active.

TABLE 3.4(a)

CHARACTERISTICS OF OPTIMIZATION PROBLEMS CONSIDERED IN GROUP II

Problem number	Brick thickness	Economics model	Criterion function	Constraints	Type of insulation
5 and 6	0.225 m	Annual cost for problem 5 Present worth for problem 6	Brick + Equipment + Insulation + Running and maintenance cost	Upper and lower bounds on insulation thickness	Mineral wool
7 and 8	0.3375 m	Annual cost for problem 7 Present worth for problem 8	Brick + Equipment + Insulation + Running and maintenance cost	Upper and lower bounds on insulation thickness	Mineral wool
9 and 10	0.450 m	Annual cost for problem 9 Present worth for problem 10	Brick + Equipment + Insulation + Running and maintenance cost	Upper and lower bounds on insulation thickness	Mineral wool

TABLE 3.4(b)

OPTIMIZATION RESULTS OF PROBLEMS CONSIDERED IN GROUP II

Problem number	At initial design			At optimum design			% reduction in cost	Computer* time (minutes)	Cooling load at optimum design (kcal/hr)
	Design variables (m)	Penalty function	Objective function (Rs.)	Design variables (m)	Penalty function	Objective function (Rs.)			
5	$X_1 = 0.01$	26,845.1	16,581.9	X_1=0.0852	8,525.3	8,524.2	47.0	8	5,421.3
6	$X_1 = 0.01$	2,49,721.6	1,42,561.7	X_1=0.0993	75,566.8	75,565.5	26.4	8	4,765.8
7	$X_1 = 0.01$	25,615.6	15,352.5	X_1=0.0703	9,929.4	9,928.2	35.3	10	6,322.5
8	$X_1 = 0.01$	2,54,562.3	1,48,792.5	X_1=0.0939	78,959.9	78,958.9	41.5	10	4,983.6
9	$X_1 = 0.01$	23,586.6	13,323.5	X_1=0.0737	9,108.3	9,107.2	31.6	10	5,451.9
10	$X_1 = 0.01$	2,31,018.1	1,40,754.9	X_1=0.0885	82,353.3	82,352.3	37.0	10	4,758.0

* These computational times refer to the times taken on IBM 7044 computer.

TABLE 3.5

OPTIMIZATION RESULTS OF THE PROBLEM CONSIDERED IN GROUP III

Problem number	At initial design			At optimum design			% reduction in cost	Computer time	Cooling load at optimum design (kcal/hr)
	Design variables	Penalty function	Objective function (Rs.)	Design variables	Penalty function	Objective function (Rs.)			
11	$X_1 = 0.01$ m $X_2 = 0.15$	25,076.8	13,747.1	X_1=0.0779m X_2=0.1077*	6,988.0	6,980.4	49.2	25 minutes on IBM 7044 computer	5,070.9

* The constraint corresponding to absorptance became active.

TABLE 3.6

OPTIMIZATION RESULTS OF THE PROBLEMS CONSIDERED IN GROUP IV

Problem number	At initial design			At optimum design			% reduction in cost	Computer time	Cooling load at optimum design (kcal/hr)
	Design variables (m)	Penalty function	Objective function (Rs.)	Design variables (m)	Penalty function	Objective function (Rs.)			
12	$X_1 = 0.05$ $X_2 = 0.05$ $X_3 = 0.05$ $X_4 = 0.05$ $X_5 = 0.05$	1,50,577.0	1,07,243.6	X_1=0.0947 X_2=0.0765 X_3=0.0831 X_4=0.0815 X_5=0.0756	95,878.3	95,554.9	10.90	150 minutes on IBM 7044 computer	68,550.3
13	$X_1 = 0.05$ $X_2 = 0.05$ $X_3 = 0.05$ $X_4 = 0.05$ $X_5 = 0.05$	1,50,728.4	1,07,839.5	X_1=0.0964 X_2=0.0772 X_3=0.0836 X_4=0.0856 X_5=0.0866	96,185.3	95,973.1	11.04	150 minutes on IBM 7044 computer	67,381.2
14	$X_1 = 0.05$ $X_2 = 0.05$ $X_3 = 0.05$ $X_4 = 0.05$ $X_5 = 0.05$	1,50,399.7	1,07,066.3	X_1=0.0946 X_2=0.0751 X_3=0.0770 X_4=0.0828 X_5=0.0803	95,748.1	95,424.3	10.80	150 minutes on IBM 7044 computer	68,461.8

TABLE 3.7

RESULTS OF OPTIMIZATION FOR THE PROBLEM CONSIDERED IN GROUP V

Problem number	At initial design			At optimum design*			% reduction in cost	Computer time	Cooling load at optimum design (kcal/hr)
	Design variables (m)	Penalty function	Objective function (Rs.)	Design variables (m)	Penalty function	Objective function (Rs.)			
15	$X_1 = 0.05$ $X_2 = 0.05$ $X_3 = 0.05$ $X_4 = 0.05$ $X_5 = 0.05$ $X_6 = 0.80$ $X_7 = 0.50$	2,40,512.4	1,83,055.9	$X_1=0.0950$ $X_2=0.0760$ $X_3=0.0837$ $X_4=0.0815$ $X_5=0.0715$ $X_6=0.667721$ $X_7=0.458385$	85,850.2	85,845.7	53.1	240 minutes on IBM 7044 computer	62,401.5

* The constraint corresponding to upper bound on the height of the warehouse became active.

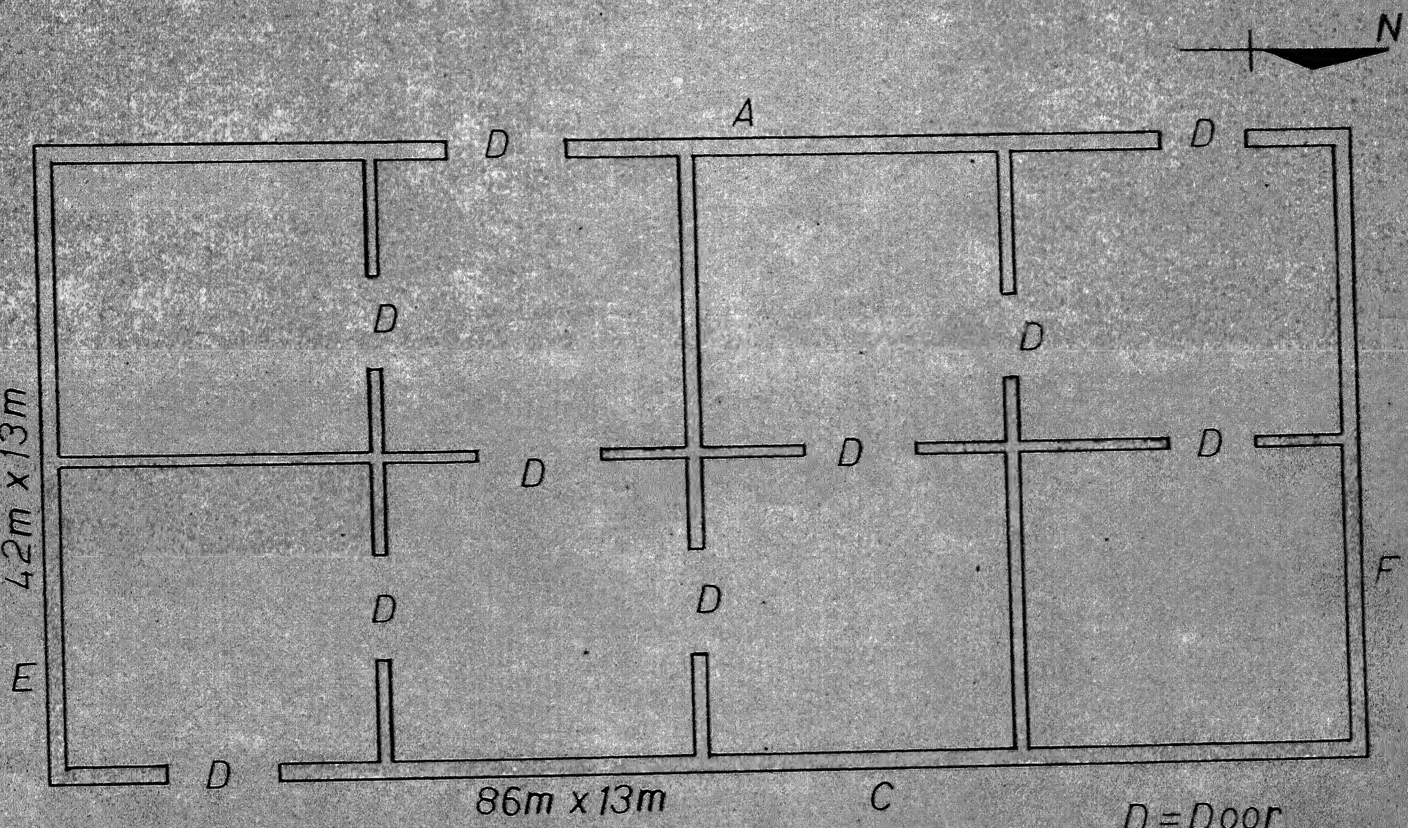
TABLE 3.8

OPTIMIZATION RESULTS OF THE PROBLEMS CONSIDERED IN GROUP VI

Problem number	At initial design: Design variables (m)	At initial design: Penalty function	At initial design: Objective function	At optimum design: Design variables (m)	At optimum design: Penalty function	At optimum design: Objective function	% reduction in cost	Computer time	Cooling load at optimum design (kcal/hr)
16*	X_1 = 0.05 X_2 = 0.05 X_3 = 0.05 X_4 = 0.05 X_5 = 0.05 X_6 = 0.80 X_7 = 0.50	3,71,605.6	Rs. 2,91,428.2	X_1=0.0912 X_2=0.0722 X_3=0.0790 X_4=0.0781 X_5=0.0721 X_6=0.691341 X_7=0.442426	2,79,949.5	Rs. 2,79,661.1	4.13	240 minutes on IBM 7044 computer	3,27,180.3
17	X_1 = 0.05 X_2 = 0.05 X_3 = 0.05 X_4 = 0.05 X_5 = 0.05 X_6 = 0.80 X_7 = 0.80	7,82,910.6	3,57,031.2 (kcal/hr)	X_1=0.1491** X_2=0.1264 X_3=0.1423** X_4=0.1298 X_5=0.1434** X_6=0.7706 X_7=0.44138	3,05,730.0	3,03,331.2 (kcal/hr)	8.30	108 minutes on IBM 370/155 computer	2,95,310.0

*Constraint corresponding to upper bound on the height of the warehouse became active.

**Constraints corresponding to insulation thicknesses for the roof and two walls became active.



(Note :- Interior walls up to a height of 10m, external walls are up to ceiling)

FIG. 3.1(a) PLAN OF THE PROPOSED REFRIGERATED WAREHOUSE.

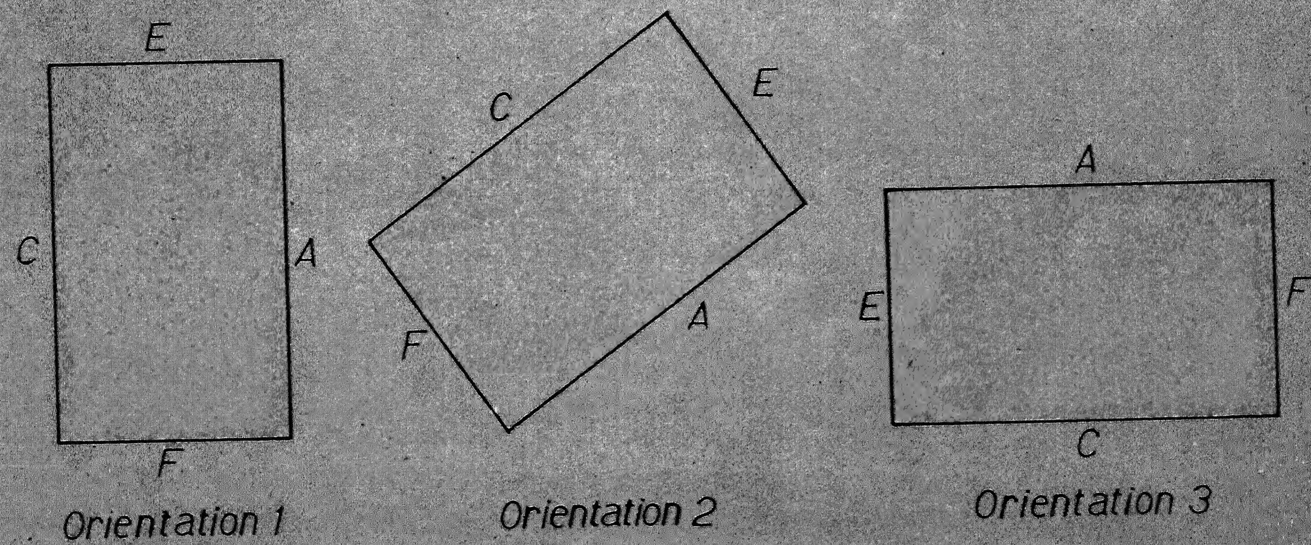


FIG. 3.1(b) PLAN OF THE WEREHOUSE SHOWING THE THREE ORIENTATIONS.

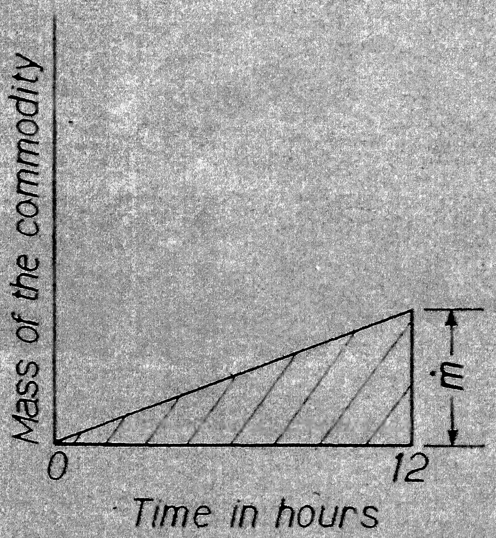


FIG. 3.2 (a) ASSUMED VARIATION OF COMMODITY LOADING IN A DAY IN WARE-HOUSE

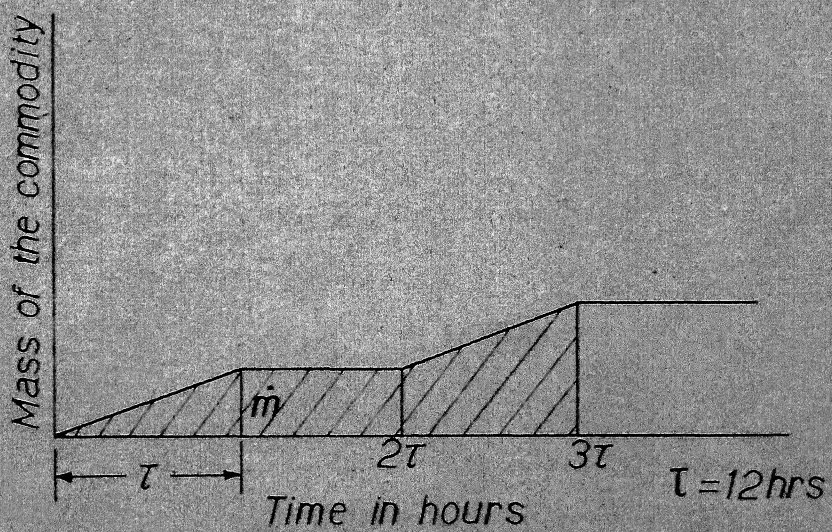


FIG. 3.2 (b) VARIATION OF COMMODITY MASS OVER THE TOTAL LOADING TIME n PERIODS.

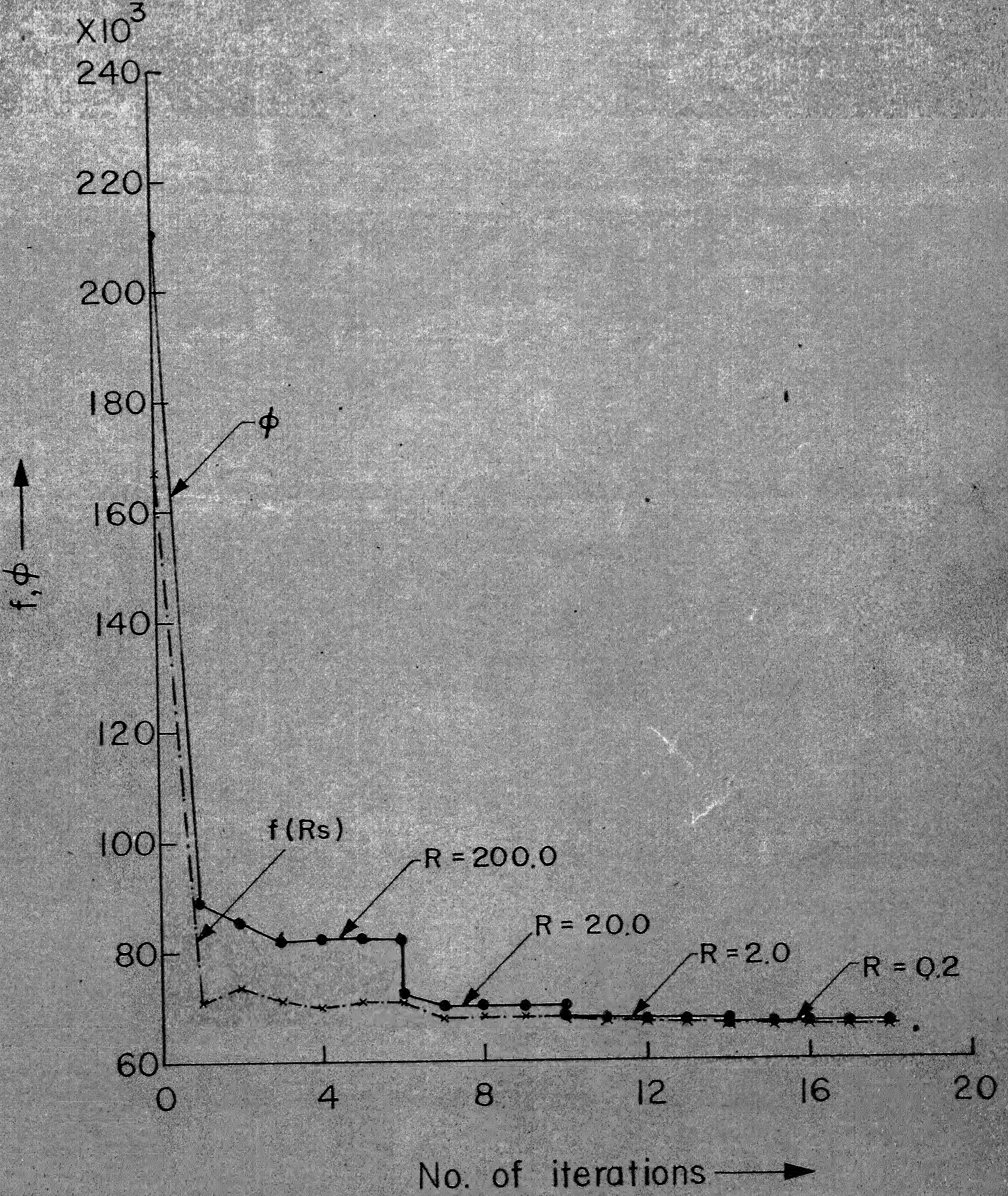


Fig. 3.3 Progress of optimization procedure for problem I

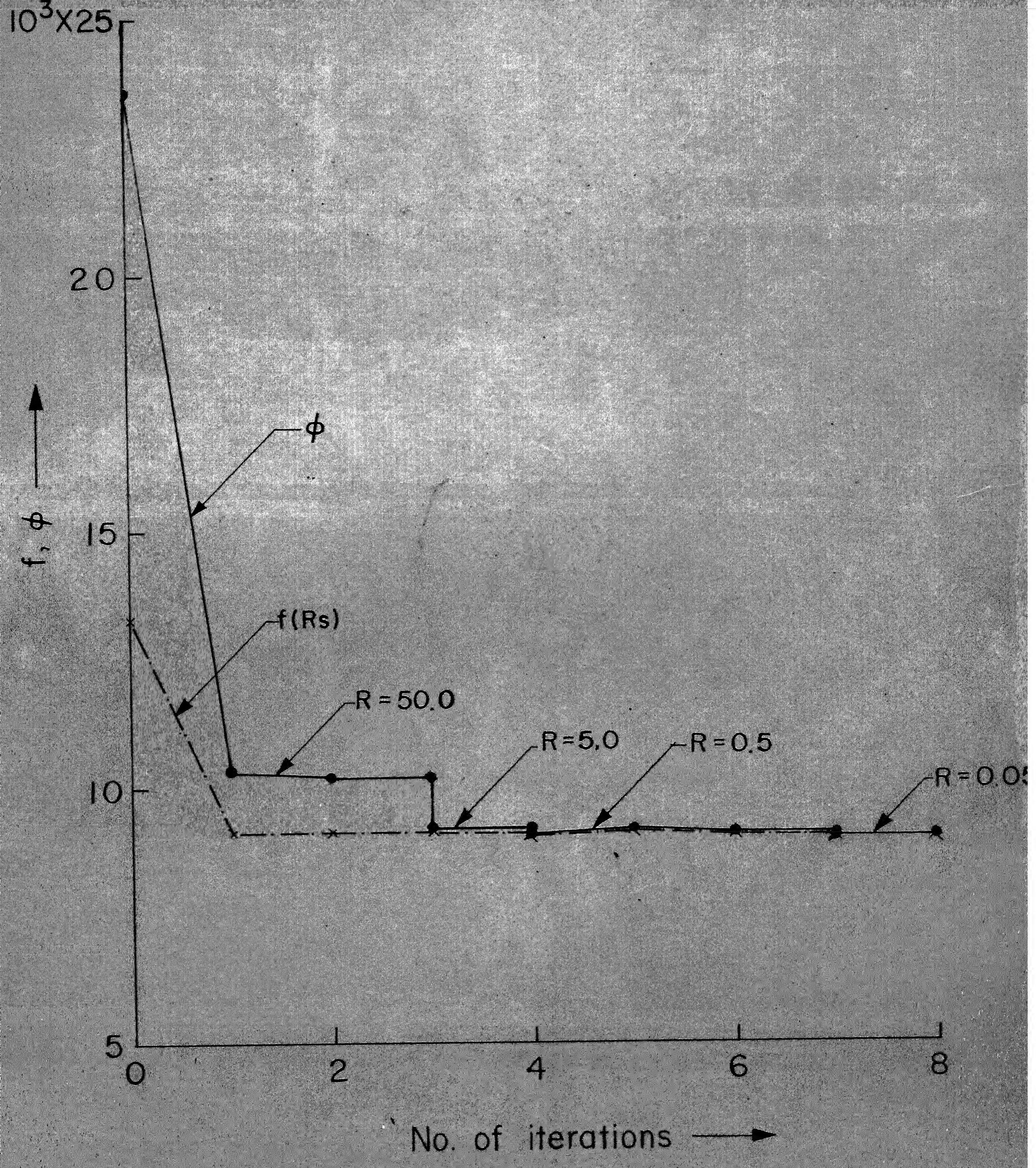


Fig. 3.4 Progress of optimization procedure for problem 9

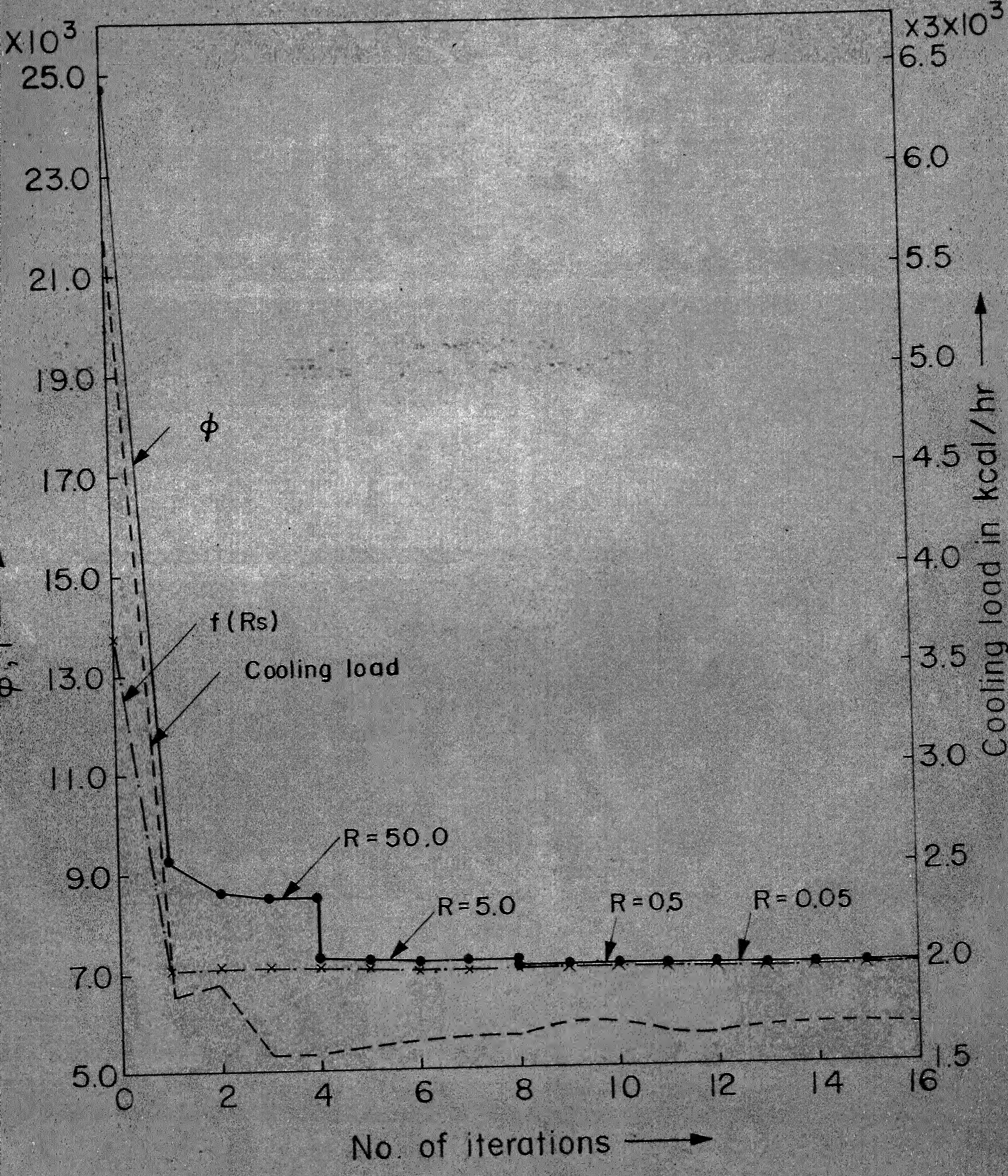


Fig. 3.5 Progress of optimization procedure for problem II

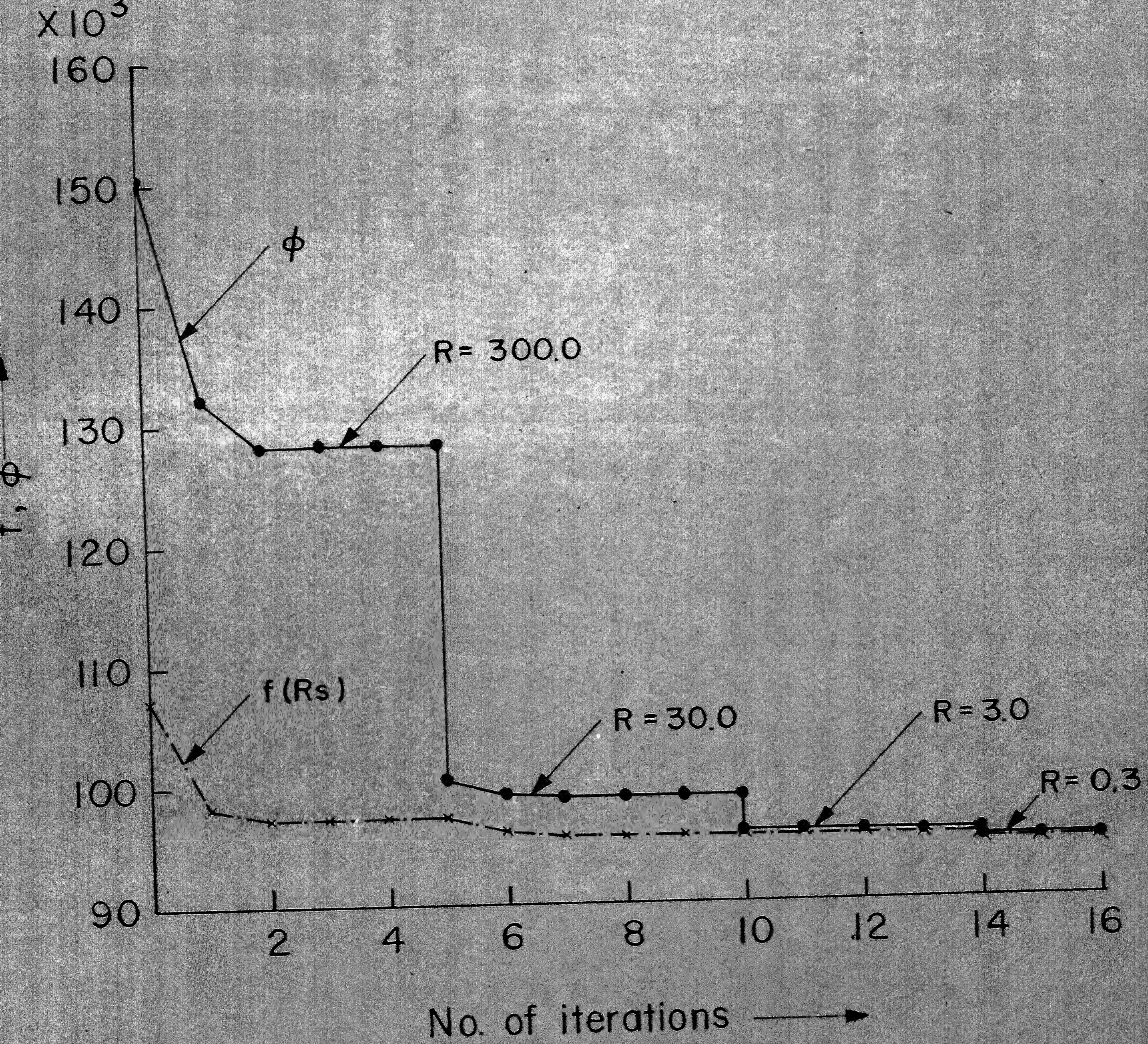


Fig.3.6 Progress of optimization procedure for problem 12

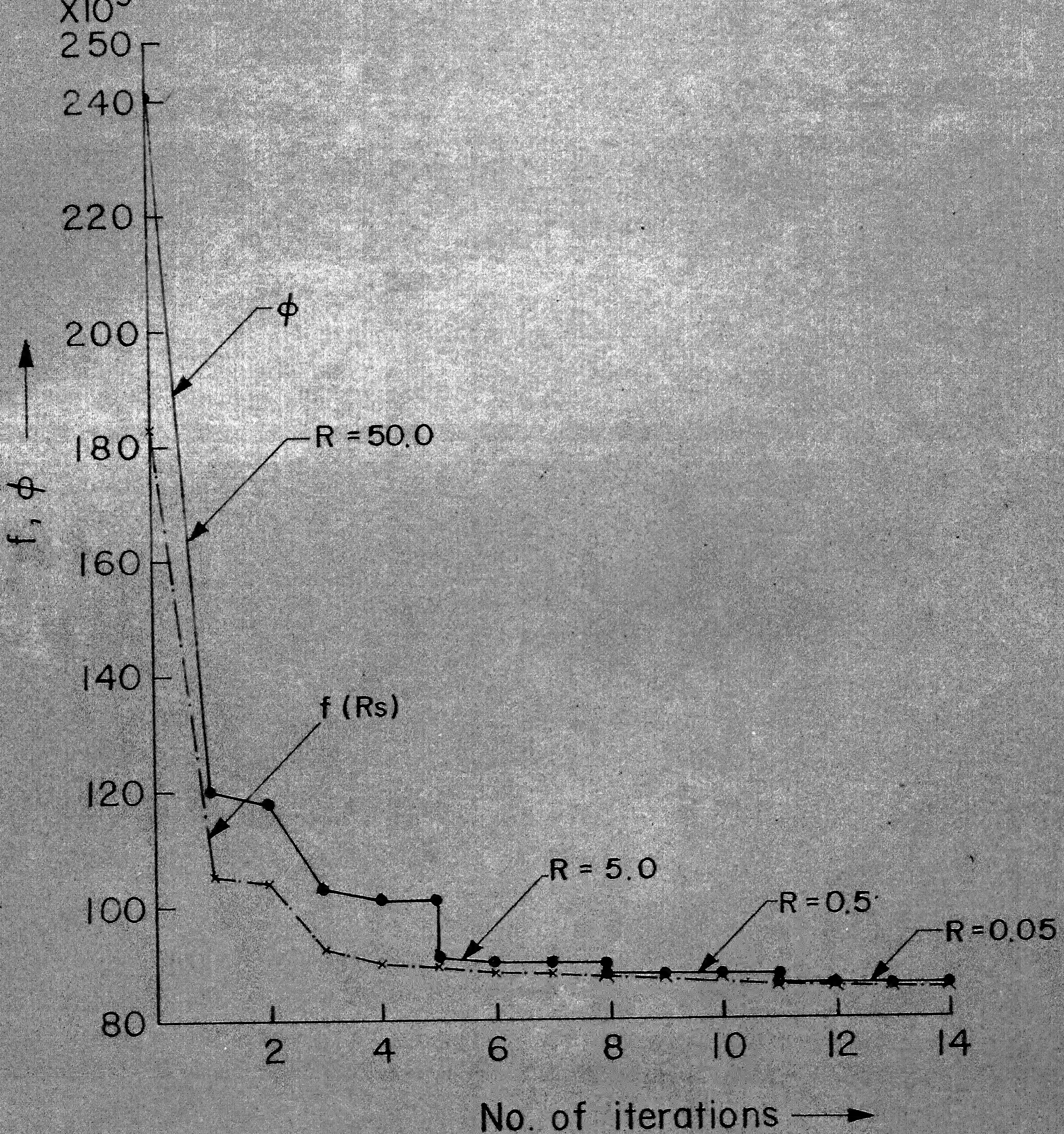


Fig.3.7 Progress of optimisation procedure for problem 15

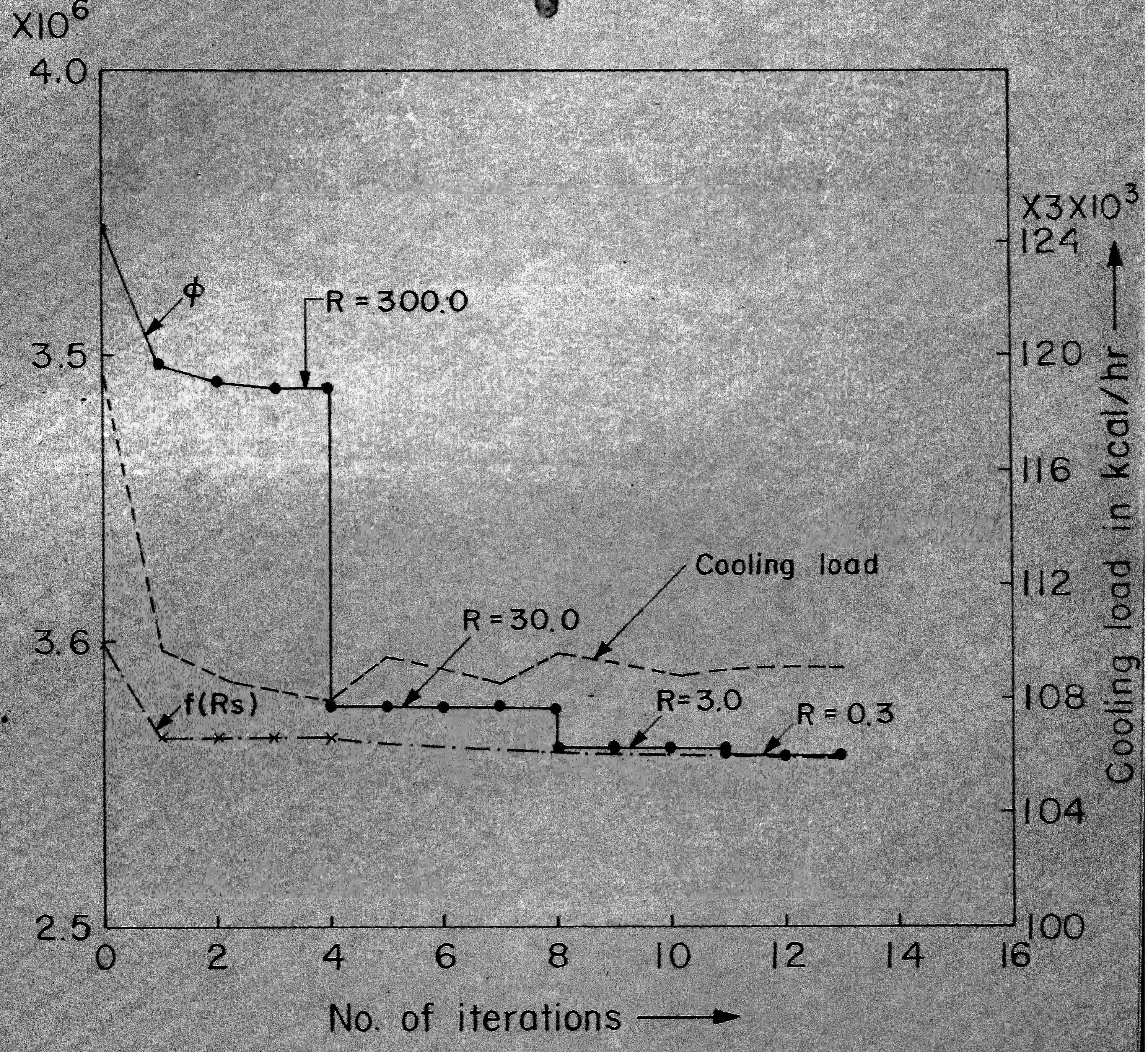


Fig.3.8 Progress of optimization procedure for problem 16

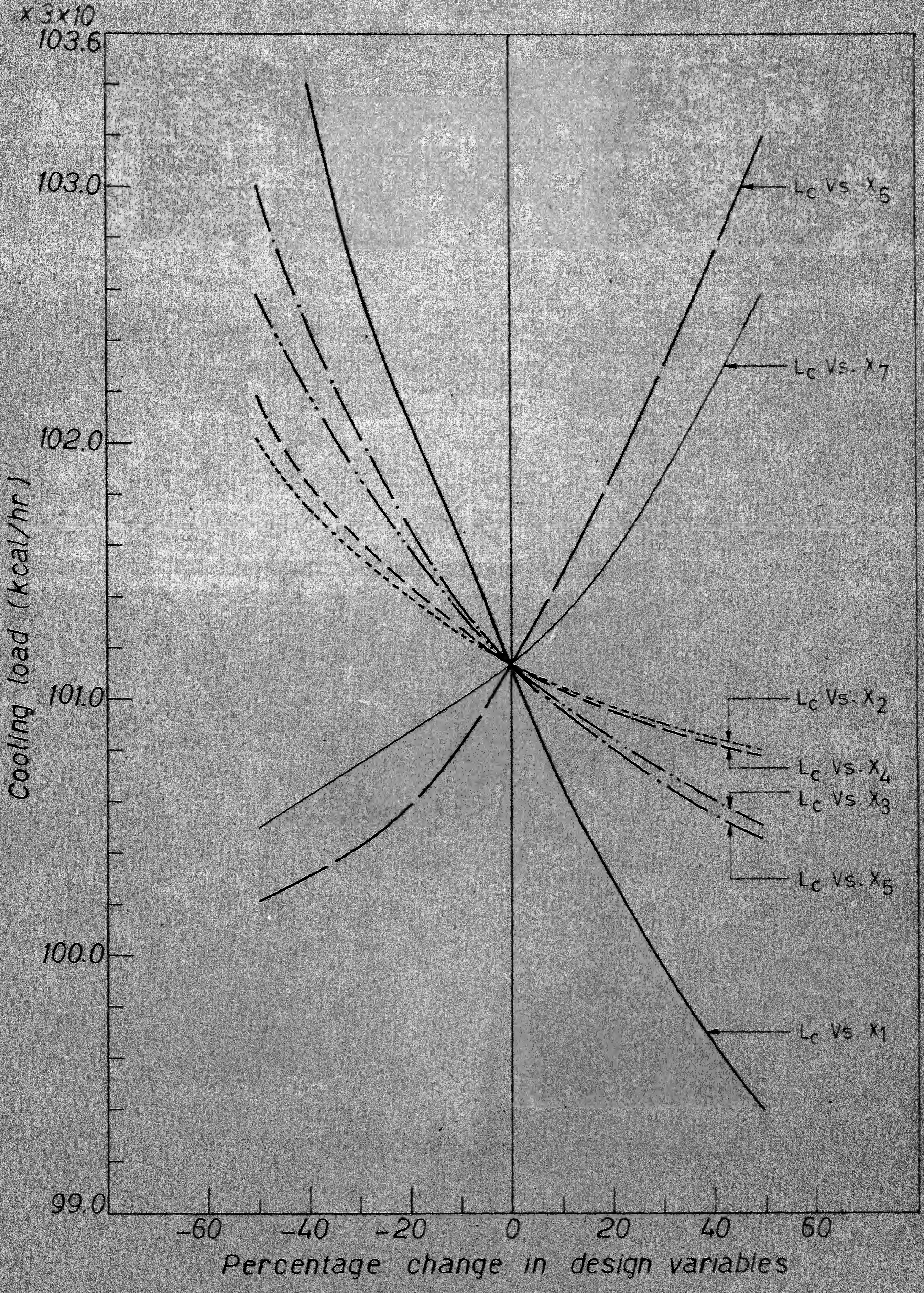


FIG. 3.9 SENSITIVITY ANALYSIS OF COOLING LOAD.

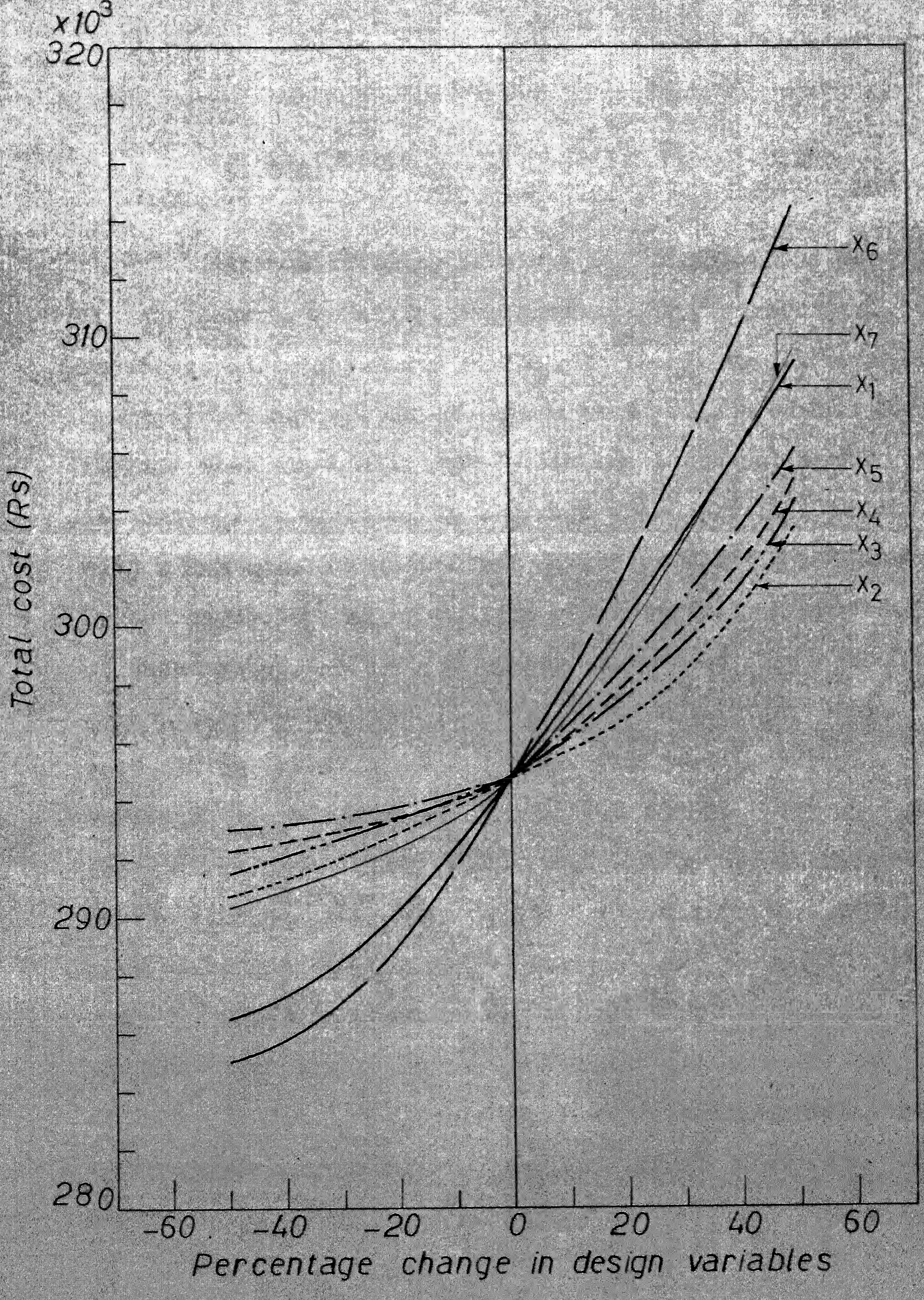


FIG. 3.10 SENSITIVITY ANALYSIS OF TOTAL COST.

CHAPTER 4

OPTIMUM DESIGN OF AIR-CONDITIONED BUILDINGS

Air-conditioning involves the simultaneous control of temperature, humidity, air motion, air distribution, dust, bacteria, odour and toxic gases. Of these, the temperature, humidity and air motion are the most important ones. This implies that air-conditioning requires both cooling as well as heating. A building is a system which provides people with a comfortable environment. Human comfort is influenced by the psychological as well as physiological characteristics of human being.. There is no precise method of stating which thermal environmental conditions will affect the comfort feeling of a human being. The most important variables which influence thermal comfort are: (i) activity level (heat production in the body), (ii) thermal resistance of the clothing (clo-value), (iii) air temperature, (iv) mean radiant temperature, (v) relative air velocity, and (vi) water vapour pressure in ambient air. ASHRAE has studied human reactions to environment and gave the concept of an empirical index called "effective temperature", which is an index expressing the combined effect of all the factors mentioned above on human comfort.

Fanger [57] has generalized the physiological basis of comfort so that for any activity, comfort can be predicted analytically in terms of the environmental parameters. Fanger's comfort equation is of the form:

$$\frac{M}{A_{Du}}(1-\eta) - 0.35[43 - 0.061\frac{M}{A_{Du}}(1-\eta) - p_a] -$$
$$[0.42\frac{M}{A_{Du}}(1-\eta) - 50] - 0.0023\frac{M}{A_{Du}}(44 - p_a) -$$
$$0.0014\frac{M}{A_{Du}}(34 - t_a) = 3.4 \times 10^{-8} f_{cl}[(t_{cl} + 273)^4 -$$
$$(t_{mrt} + 273)^4] + f_{cl}h_c(t_{cl} - t_a) \qquad (4.1)$$

with

$$t_{cl} = 35.7 - 0.032\frac{M}{A_{Du}}(1-\eta) - 0.18\, I_{cl}\{\frac{M}{A_{Du}}(1-\eta) -$$
$$0.35[43 - 0.061\frac{M}{A_{Du}}(1-\eta) - p_a] - 0.42[\frac{M}{A_{Du}}(1-\eta)$$
$$- 50] - 0.0023\frac{M}{A_{Du}}(44 - p_a) - 0.0014\frac{M}{A_{Du}}(34 - t_a)\} \qquad (4.2)$$

and

$$h_c = \begin{cases} 2.05(t_{cl}-t_a)^{0.25} & \text{for } 2.05(t_{cl}-t_a)^{0.25} > 10.4\sqrt{V} \\ 10.4\sqrt{V} & \text{for } 2.05(t_{cl}-t_a)^{0.25} < 10.4\sqrt{V} \end{cases} \qquad (4.3)$$

where M = metabolic rate

A_{Du} = DuBois area (the surface area of the nude body (m^2))

η = mechanical efficiency

p_a = pressure of water vapour in ambient air

t_a = air temperature (°C)

f_{cl} = the ratio of the surface area of the clothed body to the surface area of the nude body

t_{cl} = mean temperature of outer surface of clothed body (°C)

t_{mrt} = mean radiant temperature (°C)

h_c = convective heat transfer coefficient (kcal/m^2 hr °C)

V = relative air velocity (m/sec) and

I_{cl} = thermal resistance of the clothing.

Using the comfort Eq. (4.1), it is possible for any type of clothing (clo) and type of activity to calculate all reasonable combinations of air temperature, air humidity, mean radiant temperature and relative velocity which will create optimal thermal comfort for persons in steady state conditions. The solution of Eq. (4.1) is also available in the form of comfort diagrams.

However ASHRAE comfort standard 55-66 states thermal comfort as the condition of mind which expresses satisfaction with the thermal environment. It recommends the following conditions:

dry bulb temperature	=	73 - 77°F
relative humidity	=	20 to 60%
air velocity	=	45 fpm maximum and 10 fpm minimum
t_{mrt}	=	70 to 80°F

The common practice is to design comfort air-conditioning for summer to provide space conditions of about 24°C dry bulb temperature and 50% relative humidity. In fact Nevins and Gagge [58] have given new comfort charts which are primarily the combination of dry bulb temperature and water vapour pressure and Gagge and Nishi [59] have given physical indices for thermal environment.

The inside design conditions are dictated by comfort charts while the outside conditions are taken as stated earlier in Chapter 3.

4.1 Computation of Cooling Load

The various contributions to the cooling load are:

(i) Load from solar radiation, temperature difference of glass areas, exterior walls and roof.

(ii) Load due to heat gain through interior partitions, ceilings and floors of adjoining unconditioned space.

(iii) Load due to heat sources within the conditioned space such as occupants, lighting, power, equipment and appliances.

(iv) Load from infiltration of outdoor air.

(v) Load from outdoor ventilation air.

(vi) Miscellaneous heat loads.

The method of computing heat gain from exterior walls and roof is same as the one given in Appendix A. The heat gain through the floor is neglected in air-conditioned buildings [60]. The heat gain due to contributions (iii), (iv), (v) and (vi) is computed as described in Chapter 3. The heat gain due to glass areas is discussed in section 4.1.1. The total heat gain is converted into the cooling load by using the transfer function method. While evaluating the heating load, the light and human loads are not considered. Although theoretically the heating load would be reduced when the lights are switched on (with some time lag), this reduction is not considered in the present work. The plant is assumed to run during the period of occupancy only.

4.1.1 Computation of solar heat gain through glass [1]

The heat gain through glass due to the solar radiation incident on the outer surface and the temperature difference between outdoors and indoors constitutes upto 25% of the total equipment load in the air-conditioning of many modern buildings. A procedure of estimating this component of load is presented in this section. Figure 4.1 shows the disposal of a quantity of monochromatic direct solar radiation I_λ incident upon a

single sheet of glass of thickness L. A part of the incident radiation is reflected from the front surface and the remaining part is absorbed by the glass material. Because of successive internal reflections, the reflected, absorbed and transmitted radiation quantities are given by sums of infinite series. The total monochromatic transmissivity B_λ is given as

$$B_\lambda = \frac{(1 - R_f)^2 a}{1 - R_f^2 a^2} \tag{4.4}$$

where R_f denotes the fraction of each component reflected and a indicates the fraction of each component available for absorptions. Similarly the total monochromatic reflectivity ρ_λ is given as

$$\rho_\lambda = R_f + \frac{R_f(1 - R_f)^2 a^2}{1 - R_f^2 a^2} \tag{4.5}$$

As $\alpha_\lambda = 1 - B_\lambda - \rho_\lambda$, Eqs. (4.4) and (4.5) can be used to obtain

$$\alpha_\lambda = 1 - R_f - \frac{(1 - R_f)^2 a}{1 - R_f a} \tag{4.6}$$

In order to evaluate the absorption coefficient, a, used in Eqs. (4.4), (4.5) and (4.6), it is assumed that the absorbed radiation is proportional to the intensity of incident radiation and to the length of the path of the reflected beam. Thus

$$- dI_\lambda = K_c I_\lambda \, dL' \tag{4.7}$$

and

$$a = \frac{I_{\lambda,2}}{I_{\lambda,1}} = e^{-K_c L'} \qquad (4.8)$$

where K_c is called the extinction coefficient. Since

$$L' = \frac{L}{\cos\theta} = \frac{L}{(1 - \frac{\sin^2\theta}{n^2})^{1/2}} \qquad (4.9)$$

where θ is the angle of incidence of sun's rays, and n is the index of refraction of the glass. Further the component reflectivity R_f may be found as [61]

$$R_f = \frac{1}{2}\left[\frac{\sin^2(\theta - \theta')}{\sin^2(\theta + \theta')} + \frac{\tan^2(\theta - \theta')}{\tan^2(\theta + \theta')} \right] \qquad (4.10)$$

The use of two separated sheets of glass is common in buildings. Parmelee [62] has shown that for double glass,

$$B_{\lambda 1,2} = \frac{B_{\lambda 1} B_{\lambda 2}}{1 - \rho_{\lambda 1} \rho_{\lambda 2}} \qquad (4.11)$$

and

$$\rho_{\lambda 1,2} = \rho_{\lambda 1} + \frac{B^2_{\lambda 1} \rho_{\lambda 2}}{1 - \rho_{\lambda 1} \rho_{\lambda 2}} \qquad (4.12)$$

where the subscript $\lambda 1,2$ refers to the double glass combination, $\lambda 1$ refers to the first sheet of glass considered separately, and $\lambda 2$ refers to the second sheet of glass considered separately.

In heat transmission computations, it is more useful to know the solar absorption in each glass sheet rather than the absorption for both sheets. Thus we find

$$\alpha_{\lambda,1 \text{ of } 2} = \frac{[1 - (\rho_{\lambda 1} + B_{\lambda 1})][1 - \rho_{\lambda 2}(\rho_{\lambda 1} - B_{\lambda 1})]}{1 - \rho_{\lambda 1}\rho_{\lambda 2}} \tag{4.13}$$

and

$$\alpha_{\lambda,2 \text{ of } 2} = \frac{[1 - (B_{\lambda 2} + \rho_{\lambda 2})]\, B_{\lambda 1}}{1 - \rho_{\lambda 1}\rho_{\lambda 2}} \tag{4.14}$$

Once B and α are computed, the heat gain or loss Q_g by the interior of the structure through the glass can be computed as

$$Q_g = F_s B_d I_d + B_D I_D + B_R I_R + h_i(t_{g,i} - t_i) \tag{4.15}$$

Eq. (4.15) can be simplified further as

$$Q_g = (F_s B_d I_d + B_D I_D + B_R I_R) + \frac{(F_s \alpha_d I_d + \alpha_D I_D + \alpha_R I_R)}{1 + (h_i/h_o)} + U(t_o - t_i) \tag{4.16}$$

where

$$U = \frac{1}{(\frac{1}{h_i}) + (\frac{1}{h_o})} \tag{4.17}$$

F_s = sunlit fraction of the window, dimensionless,

B_d, B_D and B_R = transmissivity of direct, diffuse and reflected solar radiation respectively,

I_R = incidence of reflected solar radiation

I_D = incidence of diffuse sky radiation

I_d = incidence of direct solar radiation upon a surface and

α_d, α_D and α_R = absorptivity of direct, diffuse and reflected radiations respectively.

The heat gain Q_g is computed from Eq. (4.16). The data for direct and diffuse solar radiations is used as available from the meteorological department and reflected radiation I_R is assumed to be negligible.

4.2 Reduction of Cooling Load

The cooling load can be reduced by a proper design of the building by providing solar shadings, reducing air leakage and providing proper insulation to exterior walls and roof. The shadings can be designed either external or internal so that the solar radiations are reduced to a minimum. The other internal loads like lights, fans and equipment can also be reduced by economizing their use and placing them at suitable places. However, for existing buildings, Guy [63] has suggested the following plan of action for optimizing the performance:

(i) Reduce the primary loads on building systems

(ii) Employ the most energy effective system predictable for the application

(iii) Maintain the system for peak effectiveness

(iv) Operate the system for minimum time periods

(v) Educate tenants and operating personnel on energy conservation programme

(vi) Retain the services of a professional engineer to survey the system and supervise the execution of a plan of action.

4.2.1 Solar shadings

The cooling load can be reduced by designing the solar shadings. It can be done in the following ways.

i) Solar shading by structural design

The shading on glass windows can be reduced by forming a canopy over the window head or by placing vertical fins at the sides of the window. Some shading can also be achieved by providing windows at suitable places in the walls. The surface finish, position and colour of the shading projection can affect its effectiveness. A high solar reflectance is desirable but the position of the surfaces must be such that the radiation is not reflected back on the glazing.

ii) Solar shading by blinds, louvres or drapes

The direct solar radiation may also be excluded by devices such as venetian, canopy or roller blinds which can be positioned to cut-off the direct radiation. In addition the sky and ground diffuse radian can be reduced to some extent depending on the design of blinds.

iii) Shading by surrounding buildings

The designer cannot control the type of surrounding buildings. However, if there are relatively tall buildings near enough to cast a shadow on to the building designed, there can be a considerable saving in the solar heat load.

4.3 Noise Control in Air-Conditioned Buildings

The creation of an accoustical environment is as important as thermal environment for comfort in any air-conditioned building design. The designer must not only establish heating, cooling, and air flow performance requirements, but also accoustic performance limits for the various components and must specify pertinent installation details. It is also possible to isolate the building structure from vibrational forces by proper installation of equipment, ducts, etc. In fact, vibrations radiate in the form of noise when the frequency exceeds that of the lower limit of hearing (25 cps). It is possible to reduce the forces transmitted to the structure to a small fraction of the unbalanced force (of equipment) by using mounts having a sufficiently large static deflection.

Indoor shading devices, particularly draperies, are effective in absorbing sound originated within the conditioned space. They have little or no effect in preventing outdoor

sounds from entering the room. For excessive internally generated sound, the usual remedy is to apply accoustical treatment to the ceiling and sometimes to add carpets as well.

4.4 Statement of the Problem

In the present work the automated optimal design of an office building whose details are given in ASHRAE fundamentals [15], is considered for illustration. However, it is assumed that apart from the structural load, other loads are also brought to a minimum level. Two problems one with the objective function as the total cost of the plant (initial running and maintenance costs) plus the cost of thermal insulation, and the second with the objective function as total load are solved. The load is taken as the weighted average of the cooling and heating loads. The weights are taken to be proportional to the number of months during which cooling or heating is required in a year. Whenever the average daily solar-air temperature falls below the desired inside temperature, then it is assumed that the building is to be heated in that month. The design day for such a month is chosen based on the minimum average solar-air temperature and the hourly temperature and solar radian values corresponding to this date of the month are taken for computations. The design day for the other months are chosen based on maximum value of the solar-air temperature as discussed in Chapter 3. The characteristics of the design days computed are given in Table 4.1 for New Delhi, India.

The plan of the office building considered for optimization is given in Figure 4.2. The other details are given below:

Occupancy : 85 office workers from 8 a.m. to 5 p.m.

Lights : 17,500 watts, fluorescent and 4000 watts tungsten

Four motors: 7.5 horse-power

Inside conditions: temperature 24 °C and relative humidity 50%

Outside conditions: as given in Table 4.1

Roof : flat, 0.1 m R.C.C. slab

Floor construction: 0.1 m concrete on ground

Fenestration: 0.85 m x 1.5 m windows of regular plate glass with light colour venetian blinds

Front doors: Two, 0.75 m x 2.1 m (wood panels)

Side doors : Two, 0.75 m x 2.1 m (wood panels)

Rear doors : Two, 0.75 m x 2.1 m (wood panels).

4.5 Problem Formulation and Numerical Results

Two design problems, one with merit function as minimization of weighted combination of heating and cooling loads and the other with merit function as the total cost, are formulated and solved as nonlinear programming problems.

Problem 1: Find the insulation thicknesses for the walls and roof of the office building for minimum load when upper and lower bounds on insulation thickness are prescribed.

The load is computed as the weighted average of cooling and heating loads. The weights are evaluated as follows:

Let the solar-air temperature remain below the desired inside temperature for m_1 months and above the desired inside temperature for the remaining $(12 - m_1)$ months in a year. Further let the heating load corresponding to each of the m_1 months be q_{h_i} and the cooling load corresponding to each of the $(12 - m_1)$ months be q_{c_i}. Then the weighted average load considered (L_r) is given by

$$L_r = \frac{m_1 \sum_i q_{h_i} + (12 - m_1) \sum_i q_{c_i}}{12} \tag{4.18}$$

Mathematically the design problem can be stated as:

$$\text{Find } \vec{X} = \left\{ \begin{array}{c} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{array} \right\}$$

$$\text{such that } f(\vec{X}) \equiv L_r \rightarrow \text{minimum}$$

$$\begin{aligned} \text{subjected to } g_1, g_2&: \quad 0.005 \leq X_1 \leq 0.10 \\ g_3, g_4&: \quad 0.005 \leq X_2 \leq 0.10 \end{aligned}$$

g_5, g_6: $0.005 \le X_3 \le 0.10$

g_7, g_8: $0.005 \le X_4 \le 0.10$

g_9, g_{10}: $0.005 \le X_5 \le 0.10$

where X_1 is the thickness of roof insulation and X_2, X_3, X_4 and X_5 are the thicknesses of insulation of walls and L_r is the weighted average of cooling and heating loads.

The results of optimization are shown in Table 4.2.

Problem 2: The second problem can be stated as follows:

$$\text{Find } \vec{X} = \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{Bmatrix}$$

$$\text{such that } f(\vec{X}) = f_1(\vec{X}) + f_2(\vec{X}) + f_3(\vec{X}) \quad \text{minimum}$$

subjected to the same constraints as given in problem 1. where $f_1(\vec{X})$ = equipment cost, $f_2(\vec{X})$ = insulation cost and $f_3(\vec{X})$ = operating plus maintenance cost. The optimization results of this problem are given in Table 4.2.

It is interesting to note from Table 4.2 that the insulation thicknesses of walls and roof at the optimum point are different in the two problems. In problem 1 (for minimization of load), maximum insulation thickness is obtained

for the roof indicating that cooling load has been dominant. In problem 2 (for minimization of cost), the roof has thinner insulation than the three exposed walls which indicates that the energy transfer during heating has been dominant. However the percentage cost and load reductions are less because the structural cooling and heating loads form only about 20% of the total load. Depending on the design criterion, the solution of the first or second problem can be taken as the optimum solution. The progress in optimization procedure is shown in Figs. 4.3 and 4.4. However, the results obtained in the present work are not compared with the ones given in reference [15] as the outside conditions are different in the present calculations.

4.6 Kuhn-Tucker Conditions

The Kuhn-Tucker conditions are applied to problem 1 at the final design point $\vec{X}^*$ at which the constraints 1 and 7 are active. The value of the vector $\vec{\xi}$ is obtained by using Eq. (3.50). For this problem the vector $\vec{E}$ and matrix [D] are obtained as

$$\vec{E} = \begin{Bmatrix} 11.83 \\ 17.95 \\ 13.63 \\ 21.72 \\ 16.49 \end{Bmatrix}$$

$$\text{and} \quad [D] = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

The vector $\vec{\xi}$ is computed as

$$\vec{\xi} = \begin{Bmatrix} \xi_1 \\ \xi_7 \end{Bmatrix} = \begin{Bmatrix} 11.83 \\ 21.72 \end{Bmatrix} > \vec{0}$$

Since both the components of $\vec{\xi}$ are positive, Kuhn-Tucker conditions are satisfied and hence the final design vector $\vec{X}^*$ is guaranteed to be a relative minimum.

4.7 Sensitivity Analysis

Figures 4.5 and 4.6 show the effect of perturbing the design variables on the positive and negative sides about the optimum point. It can be seen that the cooling load as well as the total cost is most sensitive to the insulation thickness corresponding to the roof (X_1) and least to the insulation thickness of one of the walls (X_2).

TABLE 4.1

DESIGN DAYS FOR AIR-CONDITIONED BUILDING DESIGN FOR NEW DELHI, 1967*

Month	Date	Dry bulb temperature		Solar radiation		Solar-air temperature	
		Mean value °C	Standard deviation	Mean value kcal/hr-m^2	Standard deviation	Mean value °C	Standard deviation
January	5	10.60	4.22	181.62	187.28	16.65	10.55
February	1	15.39	6.25	264.87	249.33	24.22	15.25
March	1	24.33	5.20	298.87	269.82	34.29	16.49
April	17	31.40	4.50	376.0	301.12	43.93	17.66
May	31	35.90	5.09	420.87	309.42	49.92	18.92
June	5	36.96	3.05	406.62	300.22	50.51	16.90
July	14	33.20	2.68	378.50	283.15	45.82	15.62
August	1	30.94	2.47	370.69	290.41	43.29	15.64
September	20	29.53	2.72	348.81	288.30	41.15	15.57
October	1	28.45	3.24	322.56	275.19	39.21	15.50
November	27	16.37	0.68	46.43	52.10	17.91	2.43
December	23	14.38	1.23	61.49	92.77	16.44	4.27

* The dry bulb temperature, solar radiation, humidity and air velocity values are taken corresponding to the date of the different months given above.

TABLE 4.2

RESULTS OF OPTIMIZATION

Problem number - 1

Characteristics of the problem

Objective function: Minimization of total load
Insulating material: Mineral wool
Economics model : Annual cost

At initial design			At optimum design			% reduction in objective	Computer time
Design variables (m)	Penalty function	Objective function (kcal/hr)	Design variables (m)	Penalty function	Objective function (kcal/hr)		
$X_1 = 0.025$			$X_1=0.0994^*$				
$X_2 = 0.025$			$X_2=0.0718$				
$X_3 = 0.025$	90,864.6	43,375.8	$X_3=0.0895$	39,625.1	39,307.1	8.1	180 minutes on IBM 7044 computer
$X_4 = 0.025$			$X_4=0.0941^*$				
$X_5 = 0.025$			$X_5=0.0849$				

*Active constraints: Upper bound on insulation thickness
Cooling load at optimum point : 53,610.0 kcal/hr
Heating load at optimum point : 10,680.0 kcal/hr
Total cost at optimum point : Rs. 33,330.0.

Continued...

Table 4.2 (Continued)

Problem number - 2

Characteristics of the problem

Objective function : Minimization of total cost
Insulating material: Mineral wool
Economics model : Annual cost

At initial design			At optimum design			% reduction in objective	Computer time
Design variables (m)	Penalty function	Objective function (Rs.)	Design variables (m)	Penalty function	Objective function (Rs.)		
$X_1 = 0.025$			$X_1=0.0514$				
$X_2 = 0.025$			$X_2=0.0610$				
$X_3 = 0.025$	49,186.9	33,353.6	$X_3=0.0651$	32,595.1	32,483.2	2.46	160 minutes on IBM 7044 computer
$X_4 = 0.025$			$X_4=0.0535$				
$X_5 = 0.025$			$X_5=0.0329$				

Cooling load at optimum point : 55,410.0 kcal/hr
Heating load at optimum point : 10,520.0 kcal/hr
Total cost at optimum point : Rs. 32,483.2.

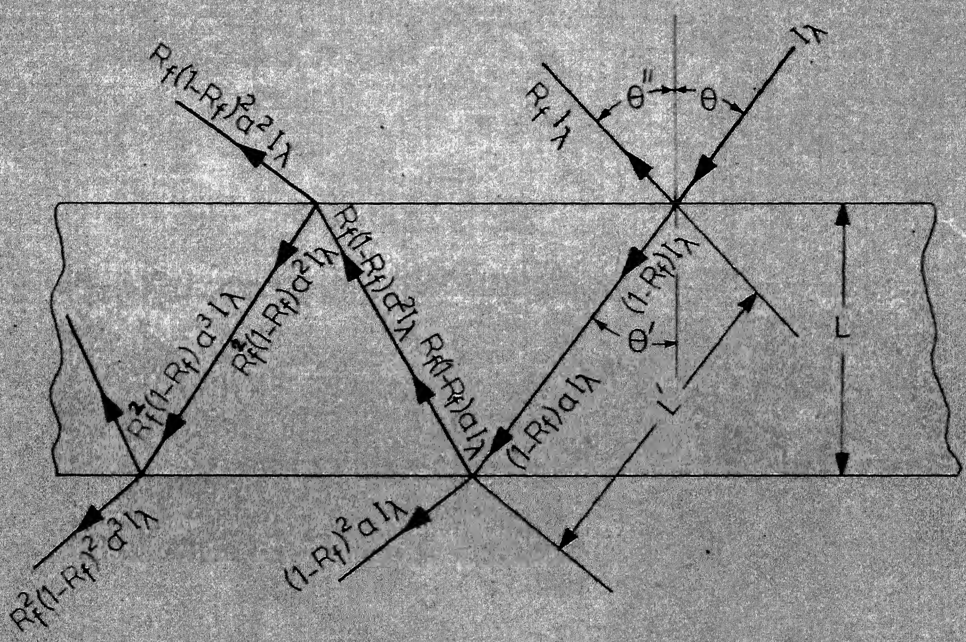


FIG.4.1 MULTIPLE REFLECTIONS OF DIRECT SOLAR RADIATION BY SINGLE SHEET OF GLASS.

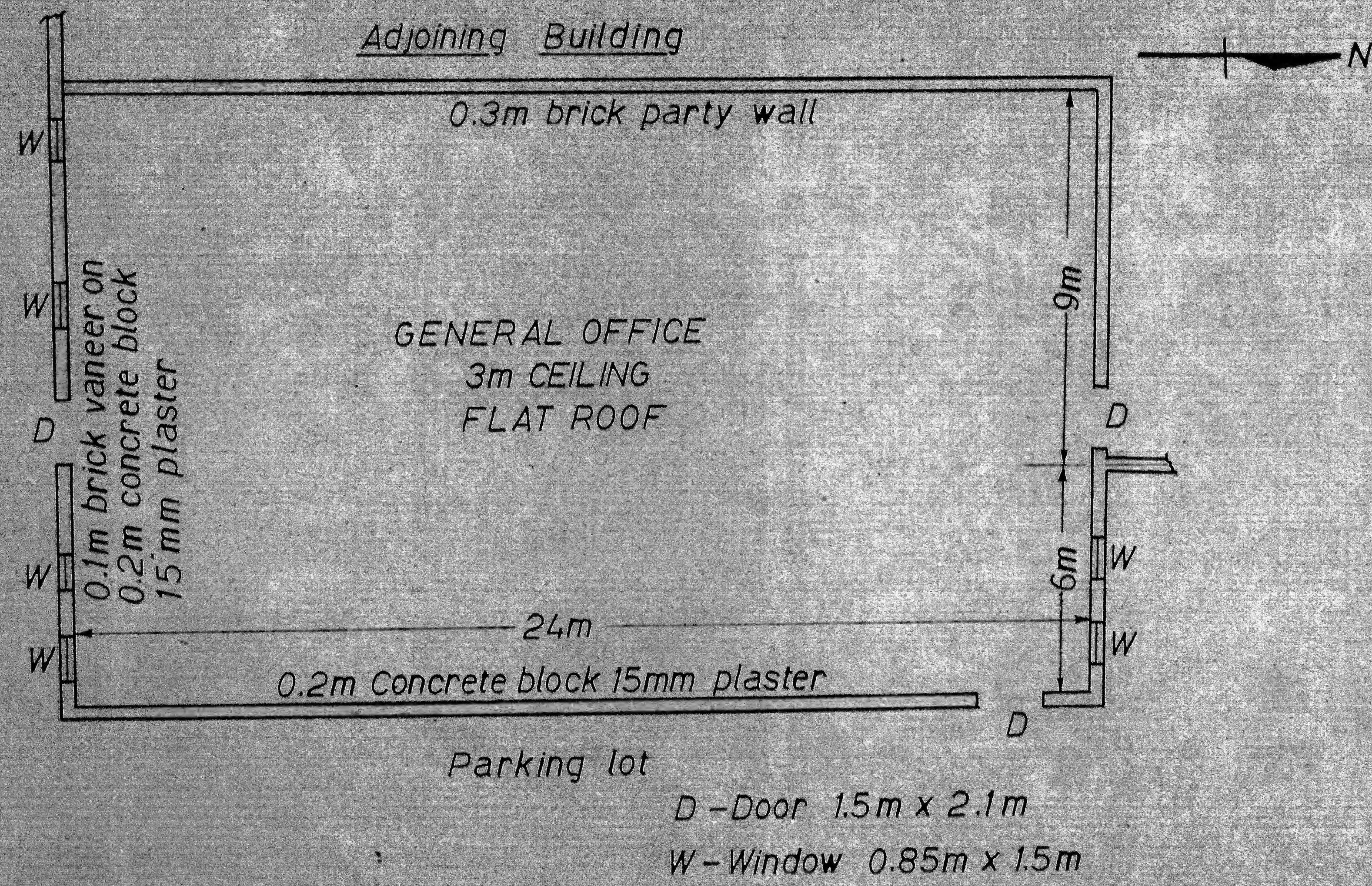


FIG. 4.2 PLAN OF ONE STOREY OFFICE BUILDING

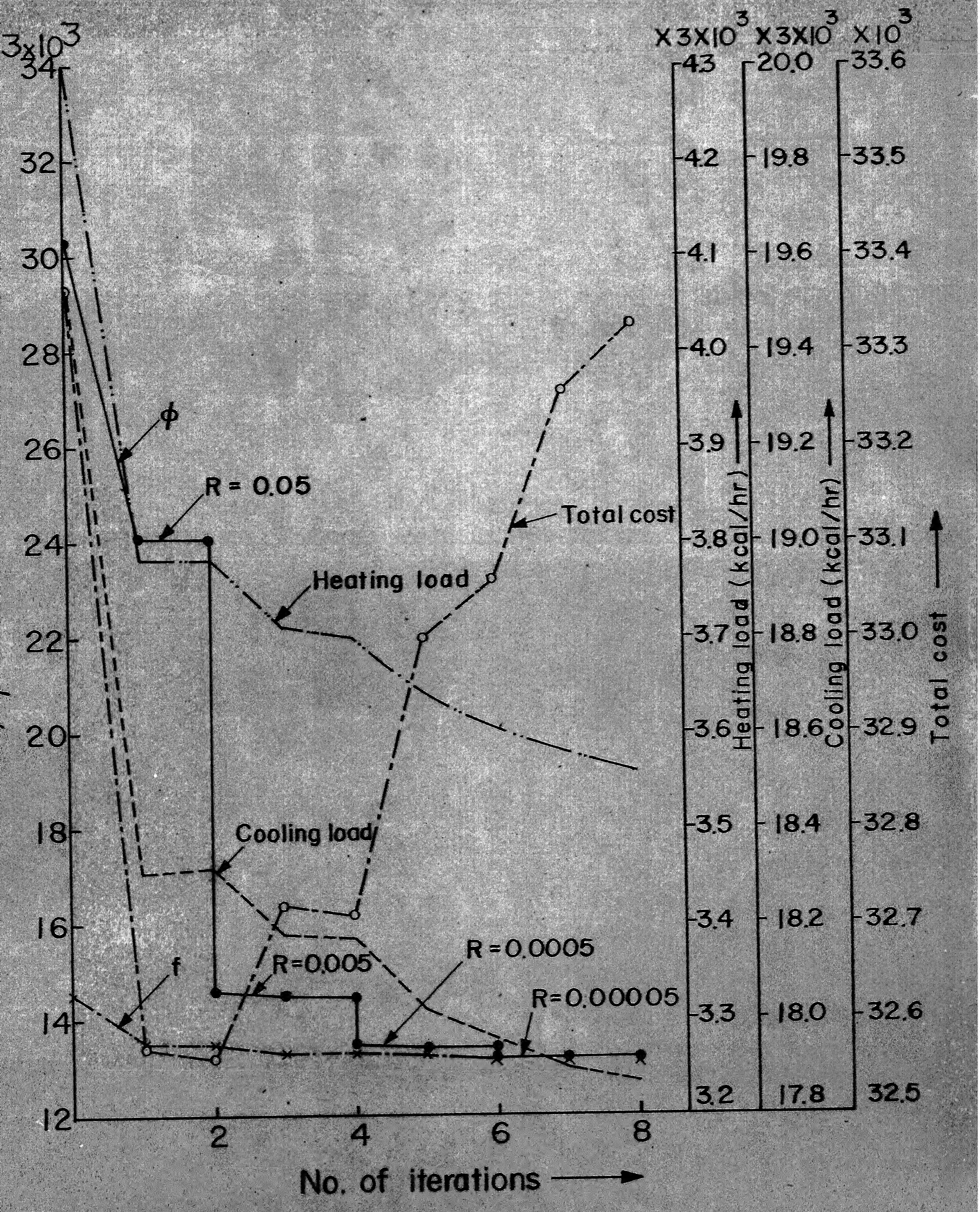


Fig. 4.3 Progress of optimization procedure for problem 1

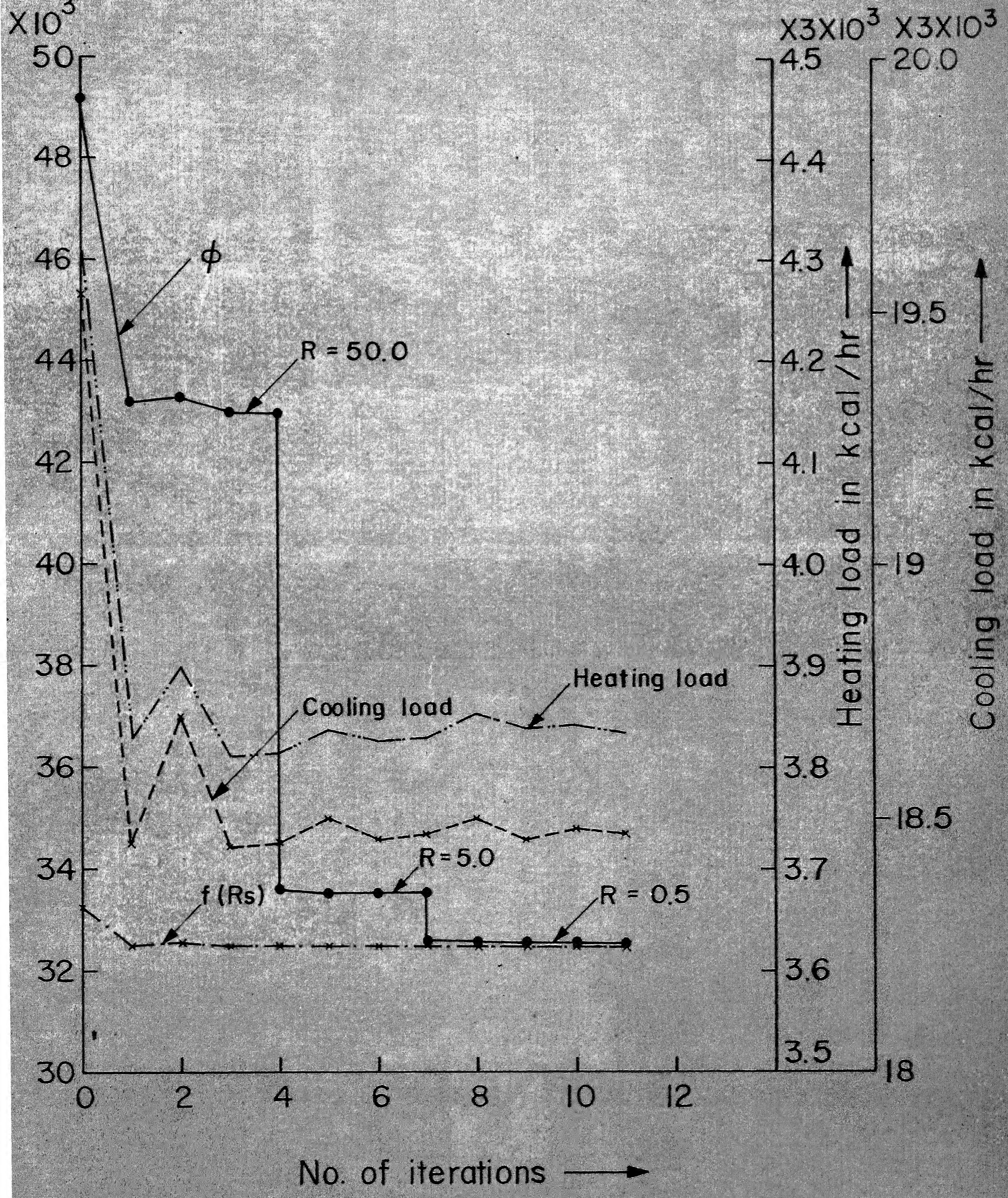


Fig.4.4 Progress of optimization procedure for problem 2

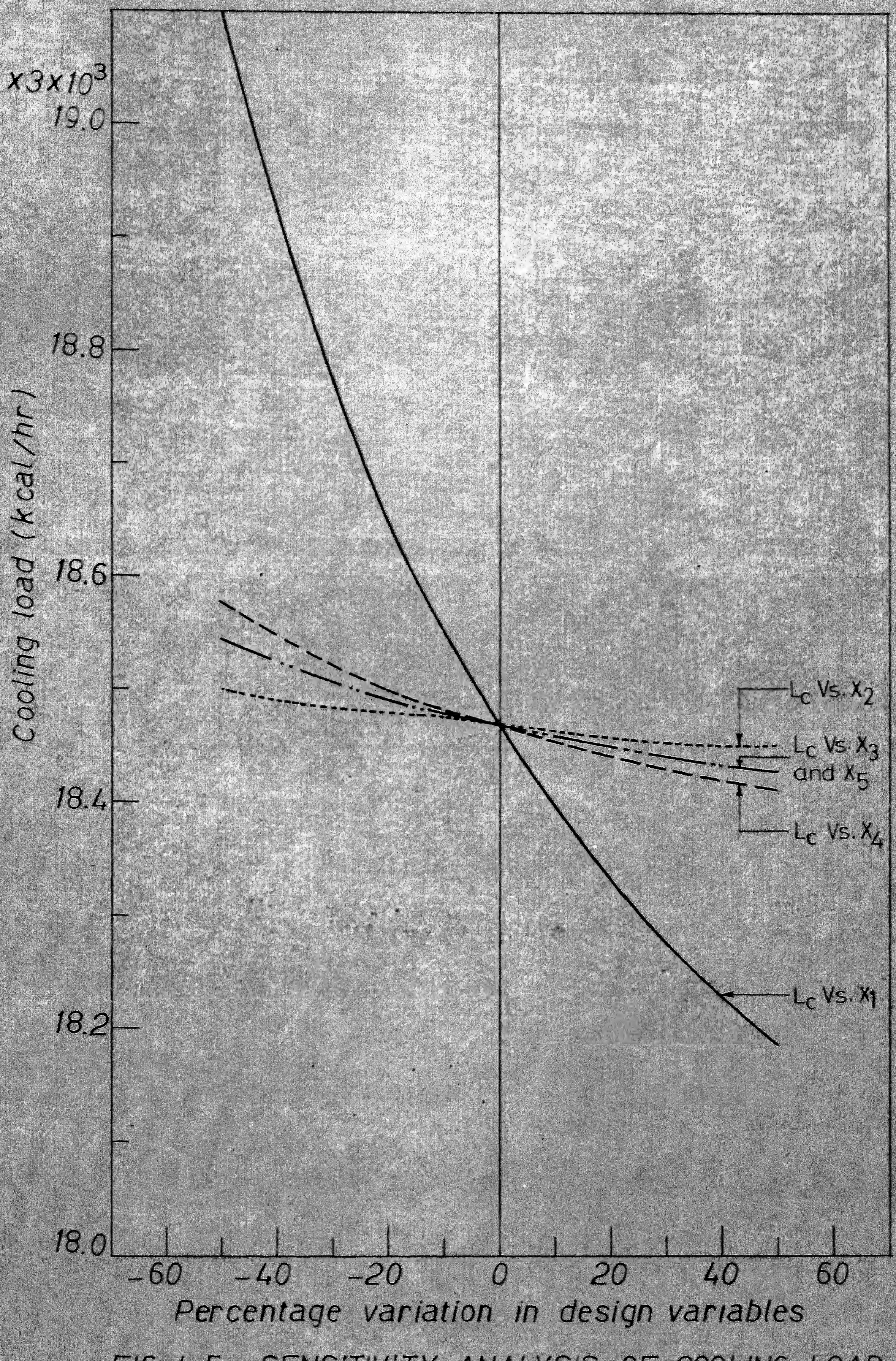


FIG. 4.5 SENSITIVITY ANALYSIS OF COOLING LOAD.

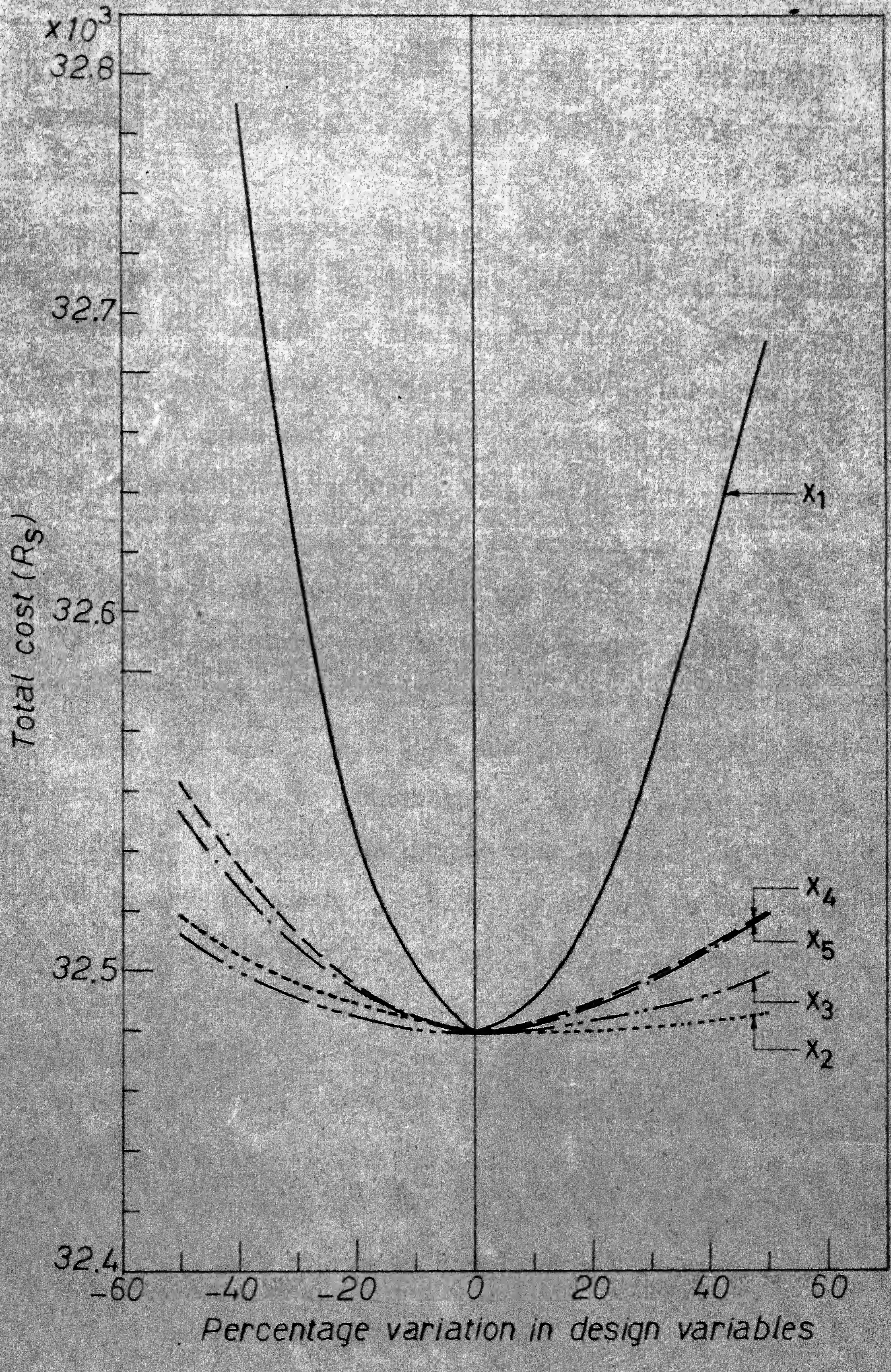


FIG. 4.6 SENSITIVITY ANALYSIS OF TOTAL COST.

CHAPTER 5

FORMULATION AND SOLUTION OF PROBABILISTIC DESIGN PROBLEM

Meteorological conditions play an important role in the design of heating or cooling systems. These conditions are beyond the control of the designer and are random in nature. A design which takes into account these random outside conditions and generate the desired inside conditions optimally has to be produced. Frequently some of the design parameters and properties of the materials used will also be random. Under these conditions, the deterministic approach for design will not be rational and a probabilistic design approach would be more suitable and realistic. Toussaint [64] has evaluated the economic loss due to discomfort caused by random fluctuations in temperature, humidity, air velocity, noise etc. in air-conditioning systems. Green and Smith [21] have discussed the use of probability theory in the design of buildings. However, the probability of occurrence of a specified load value, which varies with a variation in the outside conditions, is not computed in all the above works.

In this chapter the following problems are solved using the probabilistic design methodology:

1. Minimize the mean value of cooling load subjected to (a) upper and lower bounds on insulation thicknesses, and (b)

an upper bound on the standard deviation of the cooling load where the upper bound is taken as a fraction of the average value of the cooling load.

2. Minimize the mean value plus three times the standard deviation of the cooling load subjected to upper and lower bounds on insulation thicknesses.

3. Minimize the mean value of cooling load subjected to (a) upper and lower bounds on the insulation thicknesses, (b) a constraint that the standard deviation of the load should be less than some constant times the average value of the cooling load and (c) a restriction that the probability of realizing the maximum inside temperature less than a given value should be more than a specified value.

In these problems the probability characteristics of the cooling load which play an important role in probabilistic design have to be computed. The probability density and distribution functions of the cooling load can be found once the distribution functions of the meteorological and other parameters influencing the cooling load are known. Although this is possible theoretically, it poses a difficult problem in practice as the expression for the cooling load is quite complicated and the exact distributions of the random variables affecting the cooling load will not be known. Hence an approximate method, known as partial derivative method which makes use of only the mean values and standard

deviations of the independent random variables, is used in computing the probability characteristics of the cooling load in this work.

5.1 Design Day

The design day for the probabilistic design can be chosen based on a different criterion. It can be taken as the day of the month in which the value of mean plus three standard deviations of solar-air temperature has a maximum value. Twelve such days, each corresponding to a different month of the year, can be taken for design computations.

A second definition of the design day can be taken as the fictitious day of the month which has the hourly temperature and solar radiation values same as the average hourly temperature and solar radiations of all the days of that particular month. The standard deviations of hourly temperature and solar radiation can also be computed for each day if needed. The first definition is used in selecting the design day in this work. The characteristics of design days computed are given in Table 5.1 for New Delhi, India.

5.2 Probabilistic Description of Meteorological Data

The meteorological data for a number of years is required to compute the mean and standard deviations of solar-air temperature. In the absence of availability of hourly

data for a number of years, the values available for one year are used for the mean values and standard deviations are assumed to be certain percentage of the mean values.

Once the design days are chosen, the cooling load is to be computed on these days since the final load is taken as the weighted average of the individual loads corresponding to different months. The material properties and the building envelope parameters are also assumed as random variables.

5.3 Calculation of the Cooling Load

If all the parameters are deterministic, the heat gain can be calculated by the classical Threlkeld method [1] according to the flow chart given in Figure A.2 of Appendix A. The same procedure can be adopted for calculating the mean value of the heat gain also provided all the random parameters are taken equal to their respective mean values. However for computing the standard deviation of heat gain, the theory for finding the variance of a function of several random variables has to be used. According to the partial derivative method [22] used in this work, if F is a function of the n random variables X_1, ..., X_n, then the standard deviation of F, σ_F, is given by

$$\sigma_F^2 \simeq \left(\frac{\partial F}{\partial X_1}\Big|_{\bar{X}_1,\bar{X}_2,\ldots,\bar{X}_n}\right)^2 \sigma_{X_1}^2 + \left(\frac{\partial F}{\partial X_2}\Big|_{\bar{X}_1,\bar{X}_2,\ldots,\bar{X}_n}\right)^2 \sigma_{X_2}^2 + \ldots + \left(\frac{\partial F}{\partial X_n}\Big|_{\bar{X}_1,\bar{X}_2,\ldots,\bar{X}_n}\right)^2 \sigma_{X_n}^2 \qquad (5.1)$$

where $\left.\frac{\partial F}{\partial X_i}\right|_{\bar{X}_1,\bar{X}_2,\ldots,\bar{X}_n}$ is the partial derivative of the function F with respect to X_i evaluated at the mean values $\bar{X}_i$ of X_i and σ_{X_i} is the standard deviation of X_i $(i = 1, 2, \ldots, n)$.

5.3.1 Mean and standard deviations of cooling load

The expressions for the mean and standard deviations of cooling load can be derived as follows:

The probability characteristics (mean and standard deviation) of solar-air temperature (t_e) are given by

$$\bar{t}_e = \bar{t}_o + \bar{a}\,\frac{\bar{I}}{\bar{h}_o} \tag{5.2}$$

$$\sigma^2_{t_e} = \sigma^2_{t_o} + \left(\frac{I}{\bar{h}_o}\right)^2 \sigma^2_a + \left(\frac{a}{h_o}\right)^2 \sigma^2_I + \left(\frac{aI}{h_o^2}\right)^2 \sigma^2_{h_o} \tag{5.3}$$

where a bar represents the mean value and a sigma represents the standard deviation of the parameter and the terms in brackets are evaluated at the mean values of the variables.

The mean and standard deviations of solar radiation incident on the wall or roof per unit area I can be obtained by using the expressions given in section 3.3.3. The declination δ, total and diffused solar radiations (I_T and I_D) and absorptivity a are assumed to be random variables. The mean and standard deviations of the direction cosines (cos Z, cos W and cos S) of the solar beam are

$$\bar{Z} = \cos^{-1}[\sin L_a \sin \bar{\delta} + \cos L_a \cos \bar{\delta} \cos H] \tag{5.4}$$

$$\sigma_Z^2 = \frac{(\sin L_a \cos \bar{\delta} - \cos L_a \sin \bar{\delta} \cos H)^2}{(1 - \cos^2 \bar{Z})} \quad 2 \tag{5.5}$$

$$\bar{W} = \cos^{-1}[\cos \bar{\delta} \sin H] \tag{5.6}$$

$$\sigma_W^2 = \frac{\sin^2 \bar{\delta} \quad \sin^2 H}{(1 - \cos^2 \bar{W})} \sigma_\delta^2 \tag{5.7}$$

$$\bar{S} = \cos^{-1}[1 - \cos^2 \bar{W} - \cos^2 \bar{Z}]^{1/2} \tag{5.8}$$

$$\sigma_S^2 = \frac{(\cos^2 \bar{W} \sin^2 \bar{W} \sigma_W^2 + \cos^2 \bar{Z} \sin^2 \bar{Z} \sigma_Z^2)}{(1 - \cos^2 \bar{S}) \cos^2 \bar{S}} \tag{5.9}$$

The mean and standard deviations of the incident angle Θ can be expressed as

$$\bar{\Theta} = \cos^{-1}[l \cos \bar{Z} + m \cos \bar{W} + n \cos \bar{\delta}] \tag{5.10}$$

$$\sigma_\Theta^2 = \frac{(l^2 \sin^2 \bar{Z} \sigma_Z^2 + m^2 \sin^2 \bar{W} \sigma_W^2 + n^2 \sin^2 \delta \quad \sigma_\delta^2)}{(1 - \cos^2 \bar{\Theta})} \tag{5.11}$$

Thus the mean and standard deviations of the incident radiation I are given by

$$\bar{I} = [(\bar{I}_T - \bar{I}_D) \cos \bar{\Theta} + \bar{I}_D] \bar{a} \tag{5.12}$$

$$\sigma_I^2 = \bar{a}^2 \cos^2\bar{\Theta} \sigma_{I_T}^2 + (1-\cos\bar{\Theta})^2 \bar{a}^2 \sigma_{I_D}^2 + [(\bar{I}_T-\bar{I}_D) \cos\bar{\Theta} + \bar{I}_D]^2 \sigma_a^2 \tag{5.13}$$

The mean and standard deviation of the overall heat transfer coefficient, U, are given by

$$\bar{U} = \frac{1}{\frac{1}{\bar{h}_i} + \frac{\bar{L}}{\bar{K}} + \frac{1}{\bar{h}_o}} \tag{5.14}$$

$$\sigma_U^2 = \left(\frac{1}{h_i} + \frac{L}{K} + \frac{1}{h_o}\right)^{-4} \left[\left(\frac{1}{h_i^4}\right) \sigma_{h_i}^2 + \left(\frac{1}{K^2}\right) \sigma_L^2 + \left(\frac{L}{K^4}\right) \sigma_K^2 + \left(\frac{1}{h_o^4}\right) \sigma_{h_o}^2 \right] \tag{5.15}$$

For a composite wall, let suffix i represent the inside layer, m_1, m_2, ..., m_n, the first, second, n^{th} middle layer, and o the outside layer. Then the mean and standard deviations of $(\frac{L}{K})_{eq}$ in terms of $(\frac{L}{K})$ of individual layers can be expressed as

$$\left(\frac{\bar{L}}{\bar{K}}\right)_{eq} = \left(\frac{\bar{L}}{\bar{K}}\right)_i + \left(\frac{\bar{L}}{\bar{K}}\right)_o + \left(\frac{\bar{L}}{\bar{K}}\right)_{m_1} + \left(\frac{\bar{L}}{\bar{K}}\right)_{m_2} + \ldots + \left(\frac{\bar{L}}{\bar{K}}\right)_{m_n} \tag{5.16}$$

$$\sigma^2_{(\frac{L}{K})_{eq}} = \left(\frac{1}{K^2}\right)_i \sigma_{L_i}^2 + \left(\frac{L^2}{K^4}\right)_i \sigma_{K_i}^2 + \left(\frac{1}{K^2}\right)_o \sigma_{L_o}^2 + \left(\frac{L^2}{K^4}\right)_o \sigma_{K_o}^2 + \left(\frac{1}{K^2}\right)_{m_1} \sigma_{L_{m_1}}^2 + \left(\frac{L^2}{K^4}\right)_{m_1} \sigma_{K_{m_1}}^2 + \ldots + \left(\frac{1}{K^2}\right)_{m_n} \sigma_{L_{m_n}}^2 + \left(\frac{L^2}{K^4}\right)_{m_n} \sigma_{K_{m_n}}^2 \tag{5.17}$$

The mean and standard deviations of the coefficient $t_{e,m}$ in terms of solar-air temperature at different hours are given as

$$\bar{t}_{e,m} = \frac{1}{24} \bar{t}_{e1} + \bar{t}_{e2} + \ldots + \bar{t}_{e24} \tag{5.18}$$

$$\sigma^2_{t_{e,m}} = \frac{1}{24} [\sigma^2_{t_{e1}} + \sigma^2_{t_{e2}} + \ldots + \sigma^2_{t_{e24}}] \tag{5.19}$$

The mean and standard deviation of the product $(K\rho C)_{eq}$ and $(S_n L)_{eq}$ are given as

$$(\bar{K}\bar{\rho}\bar{C})_{eq} = \frac{1}{(\frac{\bar{L}}{\bar{K}})_{eq}} [1.1(\frac{\bar{L}}{\bar{K}})_i (\bar{K}\bar{\rho}\bar{C})_i + 1.1(\frac{\bar{L}}{\bar{K}})_{m_1} (\bar{K}\bar{\rho}\bar{C})_{m_1} + \ldots + 1.1(\frac{\bar{L}}{\bar{K}})_{m_n} (\bar{K}\bar{\rho}\bar{C})_{m_n}] + \frac{(\bar{K}\bar{\rho}\bar{C})_o}{(\frac{\bar{L}}{\bar{K}})_{eq}} [(\frac{\bar{L}}{\bar{K}})_o - 0.1(\frac{\bar{L}}{\bar{K}})_{m_1} - \ldots - 0.1(\frac{\bar{L}}{\bar{K}})_{m_n}] \tag{5.20}$$

$$\sigma^2_{(K\rho C)_{eq}} = \frac{1}{(\frac{\bar{L}}{\bar{K}})^4_{eq}} \{1.1(\bar{L}\bar{\rho}\bar{C})_i + 1.1(\bar{L}\bar{\rho}\bar{C})_{m_1} + \ldots + 1.1(\bar{L}\bar{\rho}\bar{C})_{m_n} + (\bar{K}\bar{\rho}\bar{C})_o [(\frac{\bar{L}}{\bar{K}})_o - (\frac{\bar{L}}{\bar{K}})_{m_1} x 0.1 - \ldots - 0.1(\frac{\bar{L}}{\bar{K}})_{m_n}] \}^2 \sigma^2_{(\frac{L}{K})_{eq}} +$$

$$\frac{(1.1\ \bar{\rho}\bar{C})^2}{(\frac{\bar{L}}{\bar{K}})^2_{eq}} \sigma^2_{L_i} + [\frac{(1.1\ \bar{\rho}\bar{C})_{m_1}}{(\frac{\bar{L}}{\bar{K}})_{eq}} -$$

$$\frac{(\overline{K}\overline{\rho}\overline{C})_o}{(\frac{\overline{L}}{\overline{K}})_{eq}} (\frac{0.1}{\overline{K}_{m_1}})]^2 \sigma^2_{L_{m_1}} + \ldots + [\frac{(1.1\ \overline{\rho}\overline{C})_{m_n}}{(\frac{\overline{L}}{\overline{K}})_{eq}} -$$

$$\frac{0.1(\overline{K}\overline{\rho}\overline{C})_o}{(\frac{\overline{L}}{\overline{K}})_{eq}} (\frac{1}{\overline{K}_{m_n}})]^2 \sigma^2_{L_{m_n}} + \ldots + [\frac{(\overline{\rho}\overline{C})_o}{(\frac{\overline{L}}{\overline{K}})_{eq}}]^2 \sigma^2_{L_o} +$$

$$[\frac{(\overline{K}\overline{\rho}\overline{C})_o}{(\frac{\overline{L}}{\overline{K}})_{eq}}]^2 (0.1 \frac{\overline{L}_{m_1}}{\overline{K}_{m_1}})^2 \sigma^2_{K_{m_1}} + \ldots + [\frac{(\overline{K}\overline{\rho}\overline{C})_o}{(\frac{\overline{L}}{\overline{K}})_{eq}}]^2$$

$$(0.1 \frac{\overline{L}_{m_n}}{\overline{K}_{m_n}})^2 \sigma^2_{K_{m_n}} + \frac{(1.1\overline{L}_i)^2}{(\frac{\overline{L}}{\overline{K}})^2_{eq}} (\overline{C}^2_i\ \sigma^2_{\rho_i} + \overline{\rho}^2_i\ \sigma^2_{C_i}) + \ldots +$$

$$[\frac{(1.1\overline{L}_{m_n})}{(\frac{\overline{L}}{\overline{K}})_{eq}}]^2 (\overline{C}^2_{m_n}\ \sigma^2_{m_n} + \overline{\rho}^2_{m_n}\ \sigma^2_{C_{m_n}}) +$$

$$[\frac{(\overline{K}_o)}{(\frac{\overline{L}}{\overline{K}})_{eq}} \{(\frac{\overline{L}}{\overline{K}})_o - 0.1(\frac{\overline{L}}{\overline{K}})_{m_1} - \ldots - 0.1(\frac{\overline{L}}{\overline{K}})_{m_n}\}]^2$$

$$(\overline{C}^2_o\ \sigma^2_{\rho_o} + \overline{\rho}^2_o\ \sigma^2_{C_o}) + [\frac{(\overline{\rho}\overline{C})_o}{(\frac{\overline{L}}{\overline{K}})_{eq}} \{(\frac{\overline{L}}{\overline{K}})_o - 0.1(\frac{\overline{L}}{\overline{K}})_{m_1} - \ldots -$$

$$0.1(\frac{\overline{L}}{\overline{K}})_{m_n}\} - \frac{(\overline{\rho}\overline{C})_o}{(\frac{\overline{L}}{\overline{K}})_{eq}} (\frac{\overline{L}}{\overline{K}})_o]^2 \sigma^2_{K_o} \qquad (5.21)$$

$$(\overline{S_n L})_{eq} = [\frac{\omega_n}{2} (\bar{K}\bar{\rho}\bar{C})_{eq}]^{1/2} (\frac{\bar{L}}{\bar{K}})_{eq} \tag{5.22}$$

$$\sigma^2_{(S_n L)_{eq}} = \frac{\omega_n}{2} (\bar{K}\bar{\rho}\bar{C})\ \sigma^2_{(\frac{L}{K})_{eq}} + \frac{1}{8}(\frac{\bar{L}}{\bar{K}})^2_{eq}\ \omega_n (\bar{K}\bar{\rho}\bar{C})^{-1}_{eq}\ \sigma^2_{(K\rho C)_{eq}} \tag{5.23}$$

The mean and standard deviations of the coefficients $S_n K$ are given as

$$(\overline{S_n K})_{eq} = (\frac{\omega_n}{2} (\bar{K}\bar{\rho}\bar{C})_{eq})^{1/2} \tag{5.24}$$

$$\sigma^2_{S_n K} = \frac{\omega_n}{8} (\bar{K}\bar{\rho}\bar{C})^{-1}_{eq}\ \sigma^2_{(K\rho C)_{eq}} \tag{5.25}$$

The mean and standard deviations of the coefficients Y_n, Z_n and V_n can be evaluated in a similar way.

The mean and standard deviations of the coefficients M_n, N_n and $t_{e,n}$ are given as

$$\bar{M}_n = \frac{1}{12} (\bar{t}_{e1} \cos \omega_1 \tau + \ldots + \bar{t}_{e24} \cos \omega_1 \tau) + (\bar{t}_{e1} \cos \omega_2 \tau + \ldots + \bar{t}_{e24} \cos \omega_2 \tau) \tag{5.26}$$

$$\sigma^2_{M_n} = \frac{1}{12} (\cos \omega_1 \tau + \cos \omega_2 \tau)^2\ \sigma^2_{t_{e1}} + \ldots + (\cos \omega_1 \tau + \cos \omega_2 \tau)^2\ \sigma^2_{t_{e24}} \tag{5.27}$$

$$\bar{N}_n = \frac{1}{12} (\bar{t}_{e1} \sin \omega_1 \tau + \ldots + \bar{t}_{e24} \sin \omega_1 \tau) + (\bar{t}_{e1} \sin \omega_2 \tau + \ldots + \bar{t}_{e24} \sin \omega_2 \tau) \tag{5.28}$$

$$\sigma^2_{N_n} = \frac{1}{12} (\sin \omega_1 \tau + \sin \omega_2 \tau)^2 \sigma^2_{t_{e1}} + \ldots + (\sin \omega_1 \tau + \sin \omega_2 \tau)^2 \sigma^2_{t_{e24}} \tag{5.29}$$

$$\bar{t}_{e,n} = (\bar{M}^2_n + \bar{N}^2_n)^{1/2} \tag{5.30}$$

$$\sigma^2_{t_{e,n}} = (\bar{M}^2_n + \bar{N}^2_n)^{-1} (\bar{M}^2_n \cdot \sigma^2_{M_n} + \bar{N}^2_n \cdot \sigma^2_{N_n}) \tag{5.31}$$

The mean and standard deviations of the angular displacement ϕ_n, angle λ_n and the decrement factor ψ_n are given by

$$\bar{\phi}_n = \tan^{-1} \frac{\bar{Z}_n}{\bar{Y}_n} \tag{5.32}$$

$$\sigma^2_{\phi_n} = \frac{1}{\bar{Y}^2_n} \left(\frac{\bar{Y}^2_n}{\bar{Y}^2_n + \bar{Z}^2_n}\right)^2 \sigma^2_{Z_n} + \left(\frac{\bar{Y}^2_n}{\bar{Y}^2_n + \bar{Z}^2_n}\right)^2 \frac{\bar{Z}^2_n}{\bar{Y}^4_n} \sigma^2_{Y_n} \tag{5.33}$$

$$\bar{\psi}_n = \tan^{-1} \frac{\bar{N}_n}{\bar{M}_n} \tag{5.34}$$

$$\sigma^2_{\psi_n} = \left(\frac{\bar{M}_n}{\bar{M}^2_n + \bar{N}^2_n}\right)^2 \sigma^2_{N_n} + \left(\frac{\bar{N}_n}{\bar{M}^2_n + \bar{N}^2_n}\right)^2 \sigma^2_{M_n} \tag{5.35}$$

$$\bar{\lambda}_n = \frac{\bar{V}_n}{\bar{U}} \tag{5.36}$$

$$\sigma^2_{\lambda_n} = \frac{1}{\bar{U}^2} \sigma^2_{V_n} + \frac{\bar{V}^2_n}{\bar{U}^4} \sigma^2_U \tag{5.37}$$

Finally the mean and standard deviation of the heat transfer q_i are given as

$$\bar{q}_i = \bar{U} \{[\bar{t}_{e,m} + \bar{\lambda}_1 \bar{t}_{e,1} \cos(\omega_1 \tau - \bar{\emptyset}_1 - \bar{\psi}_1) + \bar{\lambda}_2 \bar{t}_{e,2} \cos(\omega_2 \tau - \bar{\emptyset}_2 - \bar{\psi}_2) + \ldots] - t_i \} \quad (5.38)$$

$$\sigma^2_{q_i} = \{\bar{t}_{e,m} + \bar{\lambda}_1 \bar{t}_{e,1} \cos(\omega_1 \tau - \bar{\emptyset}_1 - \bar{\psi}_1) + \bar{\lambda}_2 \bar{t}_{e,2} \cos(\omega_2 \tau - \bar{\emptyset}_2 - \bar{\psi}_2) + \ldots - t_i \}^2 \sigma^2_U + \bar{U}^2 \sigma^2_{t_{e,m}} + \bar{U}^2 \bar{t}^2_{e,1} \cos^2(\omega_1 \tau - \bar{\emptyset}_1 - \bar{\psi}_1) \sigma^2_{\lambda_1} + \bar{U}^2 \bar{\lambda}^2_1 \cos^2(\omega_1 \tau - \bar{\emptyset}_1 - \bar{\psi}_1) \sigma^2_{t_{e,1}} + \bar{U}^2 \bar{\lambda}^2_1 \bar{t}^2_{e,1} \sin^2(\omega_1 \tau - \bar{\emptyset}_1 - \bar{\psi}_1) [\sigma^2_{\emptyset_1} + \sigma^2_{\psi_1}] + \ldots \quad (5.39)$$

Since the heat transfer rate through the total surface area A_i is

$$Q_i = A_i q_i \quad (5.40)$$

the mean and standard deviations of Q_i are given by

$$\bar{Q}_i = \bar{A}_i \bar{q}_i \quad (5.41)$$

$$\sigma^2_{Q_i} = \bar{q}^2_i \sigma^2_{A_i} + \bar{A}^2_i \sigma^2_{q_i} \quad (5.42)$$

To evaluate the mean and standard deviations of $t_{e,m}$, M_n and N_n, the integral is first approximated by summation over all the discrete intervals of time and then the partial derivative rule is applied. However, these quantities can also be evaluated directly as follows:

The expression for M_n is given as

$$M_n = \frac{1}{12} \int_0^{24} t_e \cos \omega_n \tau \, d\tau$$

By letting $X = M_n$ with the upper limit of integration as γ, the mean value $\overline{X}$ can be expressed as

$$\overline{X} = \overline{M}_n = \frac{1}{12} \int_0^{\gamma} \overline{t}_e \cos \omega_n \tau \, d\tau = \frac{1}{12} \overline{t}_e \int_0^{\gamma} \cos \omega_n \tau \, d\tau$$

$$= \frac{1}{12} \overline{t}_e \left[\frac{\sin \omega_n \gamma}{\omega_n} \right] \tag{5.43}$$

The expected value of $(X - \overline{X})^2$ can be written as

$$E\left[(X - \overline{X})^2\right] = E\left[\left\{\frac{1}{12} \int_0^{\gamma} t_e \cos \omega_n \tau \, d\tau - \frac{1}{12} \int_0^{\gamma} \overline{t}_e \cos \omega_n \tau \, d\tau\right\}^2\right]$$

$$= E\left[\frac{1}{144} \int_0^{\gamma} d\tau \int_0^{\gamma} d\beta \cos \omega_n \tau \cos \omega_n \beta \right.$$

$$\left. \{t_e(\tau) - \overline{t}_e\} \{t_e(\beta) - \overline{t}_e\} \right] \tag{5.44}$$

Hence the standard deviation of X, can be obtained as

$$\sigma_X^2 = \frac{1}{144} \int_0^{\gamma} d\tau \int_0^{\gamma} d\beta \cos \omega_n \tau \cos \omega_n \beta \; \kappa_{t_e,t_e}(\tau, \beta) \tag{5.45}$$

where $\kappa_{t_e,t_e}(\tau, \beta)$ is covariance function of t_e at different times τ and β.

If $t_e(\tau)$ and $t_e(\beta)$ are independent random variables, Eq. (5.45) becomes

$$\sigma_X^2 = \frac{1}{144} \int_0^{\gamma} d\tau \int_0^{\gamma} d\beta \cos \omega_n \tau \cos \omega_n \beta \, \sigma_{t_e}^2(\tau) \, \delta(\tau - \beta) \quad (5.46)$$

where δ is the Dirac delta which is equal to 1 if $\tau = \beta$ and 0 if $\tau \neq \beta$. Hence

$$\therefore \; \sigma_X^2 = \frac{1}{144} \int_0^{\gamma} d\tau \cos \omega_n \tau \, \sigma_{t_e}^2(\tau) \int_0^{\gamma} d\beta \cos \omega_n \beta \, \delta(\tau - \beta) \quad (5.47)$$

Thus σ_X can be obtained by numerical integration and the continuous values of t_e needed can be interpolated from the hourly values available. However, it requires a large computer time to evaluate the integral a number of times. Hence the integral involved is approximated by a summation (over hourly intervals) for calculating the mean and standard deviation of X.

5.3.2 Internal loads

The mean values of internal loads are evaluated as discussed in Chapter 3 and their standard deviation is assumed as 5% of the mean value. However, if the data is available, then the mean and standard deviations can be computed.

5.4 Problem Formulation and Numerical Results

In order to illustrate the applicability and effectiveness of the probability based design procedure in the

optimum design of thermal systems, three problems are formulated and solved. All these problems correspond to the design of refrigerated warehouses whose specifications are given in Chapter 3.

Problem 1: Find the insulation thicknesses for the walls and roof for the warehouse for minimum cooling load, when the upper and lower bounds on the insulation thickness are prescribed and the standard deviation of the load is constrained to be less than a fraction (k) of its mean value. Mathematically the problem can be stated as a standard optimization problem as:

$$\text{Find } \vec{X} = \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{Bmatrix}$$

$$\text{such that } f(\vec{X}) \equiv \bar{L} \rightarrow \text{minimum}$$

$$\text{subjected to } \quad 0.005 \leq X_1 \leq 0.2$$

$$0.005 \leq X_2 \leq 0.2$$

$$0.005 \leq X_3 \leq 0.2$$

$$0.005 \leq X_4 \leq 0.2$$

$$0.005 \leq X_5 \leq 0.2$$

$$\sigma_{L_c} - k\bar{L}_c \leq 0$$

where X_1 = insulation thickness for the roof,

X_2, X_3, X_4, X_5 = insulation thicknesses for the walls,

$\bar{L}_c$ = mean value of the cooling load,

σ_{L_c} = standard deviation of the cooling load, and

k = a constant.

The results of optimization obtained by using mineral wool as the insulating material are shown in Table 5.2.

The same problem is solved with the upper bound on the insulation thicknesses changed to 0.15 m (instead of 0.20 m), and the results of optimization are shown in Table 5.3.

It can be seen that the cooling load is reduced by 18.27% in the first case and 15.90% in the second case. The total annual cost in the first case is 4.6% higher than that of the second case while the load requirement is less by 3.3% in the first case. Hence, depending on the situation, either of the two solutions can be chosen. The progress of optimization procedure is shown in Figure 5.1.

Problem 2: Find the insulation thicknesses such that the mean value of the load plus three times its standard deviation is minimized, when upper and lower bounds on the insulation thicknesses are prescribed. The problem can be stated as a standard optimization problem as

$$\text{Find } \vec{X} = \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{Bmatrix}$$

such that $f(\vec{X}) = (\bar{L}_c + 3\sigma_{L_c})$ minimum

subjected to
$$0.005 \le X_1 \le 0.2$$
$$0.005 \le X_2 \le 0.2$$
$$0.005 \le X_3 \le 0.2$$
$$0.005 \le X_4 \le 0.2$$
$$0.005 \le X_5 \le 0.2$$

where X_1 = insulation thickness for the roof,

X_2, X_3, X_4, X_5 = insulation thicknesses for the walls,

$\bar{L}_c$ = mean value of the cooling load,

σ_{L_c} = standard deviation of the cooling load, and

k = constant, taken as 0.02 in the present case.

Here also mineral wool is taken as insulation material. The results of optimization are shown in Table 5.4 and Figure 5.2.

It is obvious from this figure that maximum drop in the objective function value as well as in the penalty function value takes place in first iteration. In subsequent iterations, this drop, particularly in the case of objective function, is relatively small. It was observed that the insulation thickness

gradually increases. But the effect of this increase, after a particular combination, is not very marked in reducing the load; however this increases the cost more.

Problem 3: Find the insulation thicknesses such that the mean value of the cooling load plus three times its standard deviation is minimized, when upper and lower bounds are prescribed on the insulation thicknesses and the probability of temperature of the inside surface of the wall less than a prescribed value is required to be greater than a specified value. This problem can be stated as follows:

$$\text{Find} \quad \vec{X} = \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{Bmatrix}$$

$$\text{such that} \quad f(\vec{X}) = (\bar{L}_c) \rightarrow \text{minimum}$$

$$\text{subjected to} \quad g_1(\vec{X}) = X_1 - 0.15 \leq 0$$

$$g_2(\vec{X}) = 0.005 - X_1 \leq 0$$

$$g_3(\vec{X}) = X_2 - 0.15 \leq 0$$

$$g_4(\vec{X}) = 0.005 - X_2 \leq 0$$

$$g_5(\vec{X}) = X_3 - 0.15 \leq 0$$

$$g_6(\vec{X}) = 0.005 - X_3 \leq 0$$

$$g_7(\vec{X}) = X_4 - 0.15 \leq 0$$

$$g_8(\vec{X}) = 0.005 - X_4 \leq 0$$

$$g_9(\vec{X}) = X_5 - 0.15 \leq 0$$

$$g_{10}(\vec{X}) = 0.005 - X_5 \leq 0$$

$$g_{11}(\vec{X}) = P(t_S \leq t) \geq p$$

$$g_{12}(\vec{X}) = \sigma_{L_c} - k\bar{L}_c \leq 0$$

where

X_1, X_2, X_3, X_4 and X_5 = insulation thicknesses corresponding to roof and four walls respectively,

$\bar{L}_c$ = mean value of the cooling load,

σ_{L_c} = standard deviation of the cooling load,

k = a constant,

t_S = inside maximum surface temperature of any wall or roof

t = prescribed temperature value, and

p = prescribed value of probability.

The probability constraint g_{11} can be converted into an equivalent deterministic form as suggested by Hati and Rao [65]. Let the probability constraint be written in general form as

$$P[\, g_j(\vec{X}) \leq b_j \,] \geq p_j \tag{5.48}$$

By defining a new random variable y_j as

$$y_j = F_j(\vec{X}) = b_j - g_j(\vec{X}) \tag{5.49}$$

Eq. (5.48) can be written as

$$\int_0^\infty p(y_j)dy \geq p_j \tag{5.50}$$

where $p(y_j)$ is the density function of the random variable y_j. By expanding $F_j(\vec{X})$ about the mean values of $\vec{\underline{X}}$ and retaining only the linear terms, one gets

$$y_j = F_j(\vec{X}) \simeq F_j(\vec{\underline{X}}) + \sum_i \left.\frac{\partial F_j}{\partial X_i}\right|_{\vec{\underline{X}}} (X_i - \bar{X}_i) \tag{5.51}$$

Here it is assumed that y_j is a normally distributed random variable and its mean and standard deviations are given by

$$\bar{y}_j = F_j(\vec{\underline{X}}) \tag{5.52}$$

$$\sigma_{y_j} = \left[\sum_i \left(\left.\frac{\partial F_j}{\partial X_i}\right|_{\vec{\underline{X}}} \right)^2 \sigma_{X_i}^2 \right]^{1/2} \tag{5.53}$$

with the transformation of variable $\theta = (y_j - \bar{y}_j)/\sigma_{y_j}$ and noting that

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\, dt = 1, \tag{5.54}$$

Eq. (5.50) reduces to

$$\int_{-\frac{\bar{y}_j}{\sigma_{y_j}}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{\theta^2}{2}}\, d\theta \geq \int_{\psi_a(p_j)}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\, dt \tag{5.55}$$

where $\psi_a(p_j)$ depends on the probability level p_j. Thus

$$-\frac{\bar{y}_j}{\sigma_{y_j}} \leq \psi_a(p_j)$$

$$\text{i.e.} \qquad -\bar{y}_j - \psi_a(p_j)\,\sigma_{y_j} \leq 0 \tag{5.56}$$

Eq. (5.56) can be written as

$$g_j(\vec{X}) - \psi_a(p_j)\left[\left(\sum_i \frac{\partial F_i}{\partial X_i}\Big|_{\vec{X}}\right)^2 \sigma_{X_i}^2\right]^{1/2} \leq \bar{b}_j \tag{5.57}$$

Thus the constraint g_{11} reduces to the form of Eq. (5.57) where the value of $\psi_a(p_j)$ is obtained from standard probability (normal distribution) tables corresponding to the assigned probability level p_j.

For the inside surface temperature t_S, first the wall and roof temperatures for same insulation thickness are computed and the one (walls or roof) having the maximum temperature at any time during 24 hours is picked up. Then the partial derivatives of this temperature with respect to each of the random variables, namely, the outside temperature, solar radiation incident and diffused, wind velocity (outside film coefficient) and inside film coefficient, are obtained numerically and the constraint value is evaluated using Eq. (5.57). This type of constraint can help in checking the instantaneous maximum inside surface temperature and hence the radiations

between wall and the stored commodity etc. at any instant which may affect the stored product adversely.

By using mineral wool as insulating material, the optimization problem is solved. The results are given in Table 5.5 and the progress of optimization procedure is shown in Figure 5.3. The maximum inside surface temperature (t) was specified as 6.0°C and the probability of actual temperature exceeding 6.0°C was required to be not more than 33.33%. The inside temperature is to be maintained as 0°C. Hence it is expected that there will not be any bad effect on the commodity stored in the absence of instantaneous cooling. The final maximum surface temperature (at the optimum point) came out to be 3.8°C. In fact this type of restriction may be quite useful for commodities which are very sensitive to temperature fluctuations.

5.5 Kuhn-Tucker Conditions

The Kuhn-Tucker conditions are applied to the final design point $\vec{X}^*$ of problem 3 in which the constraints 1, 3 and 7 are active. For this problem $\vec{E}$ and [D] are found to be

$$\vec{E} = \begin{Bmatrix} 52.30 \\ 13.70 \\ 6.50 \\ 12.23 \\ 7.30 \end{Bmatrix}$$

$$\text{and } [D] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

The value of the vector $\vec{\xi}$ is computed, using Eq. (3.50), as

$$\vec{\xi} = \begin{Bmatrix} \xi_1 \\ \xi_3 \\ \xi_7 \end{Bmatrix} = \begin{Bmatrix} 52.30 \\ 13.70 \\ 12.23 \end{Bmatrix} > \vec{0}$$

Since all ξ_i are positive, Kuhn-Tucker conditions are satisfied and hence the final design vector $\vec{X}^*$ is guaranteed to be a relative minimum.

5.6 Sensitivity Analysis

Figures 5.4 and 5.5 show the effect of varying the design variables about the optimal point on the positive and negative sides by 50% in steps of 10%. It is observed that the cost as well as cooling load is most sensitive to the insulation thickness of the roof (X_1) and least to the insulation thickness of one of the walls (X_2).

TABLE 5.1

DESIGN DAYS FOR PROBABILISTIC DESIGN FOR NEW DELHI, 1967

Month	Date	Dry bulb temperature		Solar radiation		Solar-air temperature		Mean + 3*standard deviation of solar-air temperature °C
		Mean value °C	Standard deviation	Mean value kcal/hr-m^2	Standard deviation	Mean value °C	Standard deviation	
January	24	15.94	6.70	217.18	214.05	23.18	14.65	67.14
February	28	21.95	6.47	288.81	264.14	31.58	16.58	81.17
March	9	22.47	5.93	340.74	292.46	33.82	17.84	87.35
April	21	30.37	5.75	389.50	312.72	43.36	19.38	101.50
May	18	32.45	6.22	438.31	318.61	47.06	20.48	108.49
June	19	34.02	3.96	438.69	309.02	48.64	18.67	104.64
July	14	33.20	2.68	378.50	283.10	45.82	15.62	92.68
August	2	30.39	2.41	340.69	316.98	41.75	16.95	92.61
September	30	28.21	3.81	335.25	285.05	39.38	16.51	88.91
October	15	27.05	5.06	298.69	267.47	37.01	16.75	87.27
November	5	21.78	4.43	261.87	246.84	30.51	15.00	75.53
December	12	16.45	3.51	211.19	209.32	23.49	11.90	59.18

TABLE 5.2

RESULTS OF OPTIMIZATION FOR PROBLEM 1

At initial design			At optimum design			% reduction in objective (cooling load)	Computer time	Total cost at optimum point (Rs.)
Design variables (m)	Penalty function	Objective function (kcal/hr)	Design variables (m)	Penalty function	Objective function (kcal/hr)			
X_1 = 0.05			X_1 = 0.1991*					
X_2 = 0.05			X_2 = 0.1913*					
X_3 = 0.05	5,78,214.3	3,61,302.9	X_3 = 0.1593	2,95,371.6	2,93,953.5	18.27	300 minutes on IBM 7044 computer	3,10,870.0
X_4 = 0.05			X_4 = 0.1920*					
X_5 = 0.05			X_5 = 0.1661					

* Active constraint: Upper bounds on insulation thickness of roof and two walls.

TABLE 5.3

OPTIMIZATION RESULTS FOR PROBLEM 1
WITH UPPER BOUND ON INSULATION THICKNESS AS 0.15 m

At initial design			At optimum design			% reduction in objective (cooling load)	Computer time	Total cost at optimum point (Rs.)
Design variables (m)	Penalty function	Objective function (kcal/hr)	Design variables (m)	Penalty function	Objective function (kcal/hr)			
X_1 = 0.05			X_1 = 0.1494*					
X_2 = 0.05			X_2 = 0.1426*				110 minutes	
X_3 = 0.05	6,04,409.1	3,61,302.9	X_3 = 0.1382	3,05,242.8	3,03,854.7	15.9	on IBM 370/155	2,96,509.0
X_4 = 0.05			X_4 = 0.1437*				computer	
X_5 = 0.05			X_5 = 0.1421*					

* Active constraint: Upper bounds on insulation thickness of roof and three walls.

TABLE 5.4

RESULTS OF OPTIMIZATION FOR PROBLEM 2

At initial design			At optimum design			% reduction in objective	Computer time	Cooling load at optimum point (kcal/hr)
Design variables (m)	Penalty function	Objective function (kcal/hr)	Design variables (m)	Penalty function	Objective function (kcal/hr)			
$X_1 = 0.05$			$X_1 = 0.1992$*					
$X_2 = 0.05$			$X_2 = 0.1912$*				300 minutes	
$X_3 = 0.05$	5,86,908.9	3,70,234.2	$X_3 = 0.1595$	3,05,998.5	3,03,643.8	18.8	on IBM 7044	2,93,910.9
$X_4 = 0.05$			$X_4 = 0.1921$*				computer	
$X_5 = 0.05$			$X_5 = 0.1669$					

*Active constraints: Upper bounds on insulation thickness of roof and two walls.

TABLE 5.5

RESULTS OF OPTIMIZATION OF PROBLEM 3

At initial design			At optimum design			% reduction in objective (cooling load)	Computer time	Total cost at optimum point (Rs.)
Design variables (m)	Penalty function	Objective function (kcal/hr)	Design variables (m)	Penalty function	Objective function (kcal/hr)			
$X_1 = 0.05$			$X_1 =$ 0.1450*					
$X_2 = 0.05$			$X_2 =$ 0.1486*				112 minutes	
$X_3 = 0.05$	6,20,498.1	3,61,302.9	$X_3 =$ 0.1302	3,06,236.6	3,04,155.6	15.8	on IBM 370/155	2,95,763.2
$X_4 = 0.05$			$X_4 =$ 0.1465*				computer	
$X_5 = 0.05$			$X_5 =$ 0.1354					

*Active constraint: Upper bounds on insulation thickness of roof and two walls.

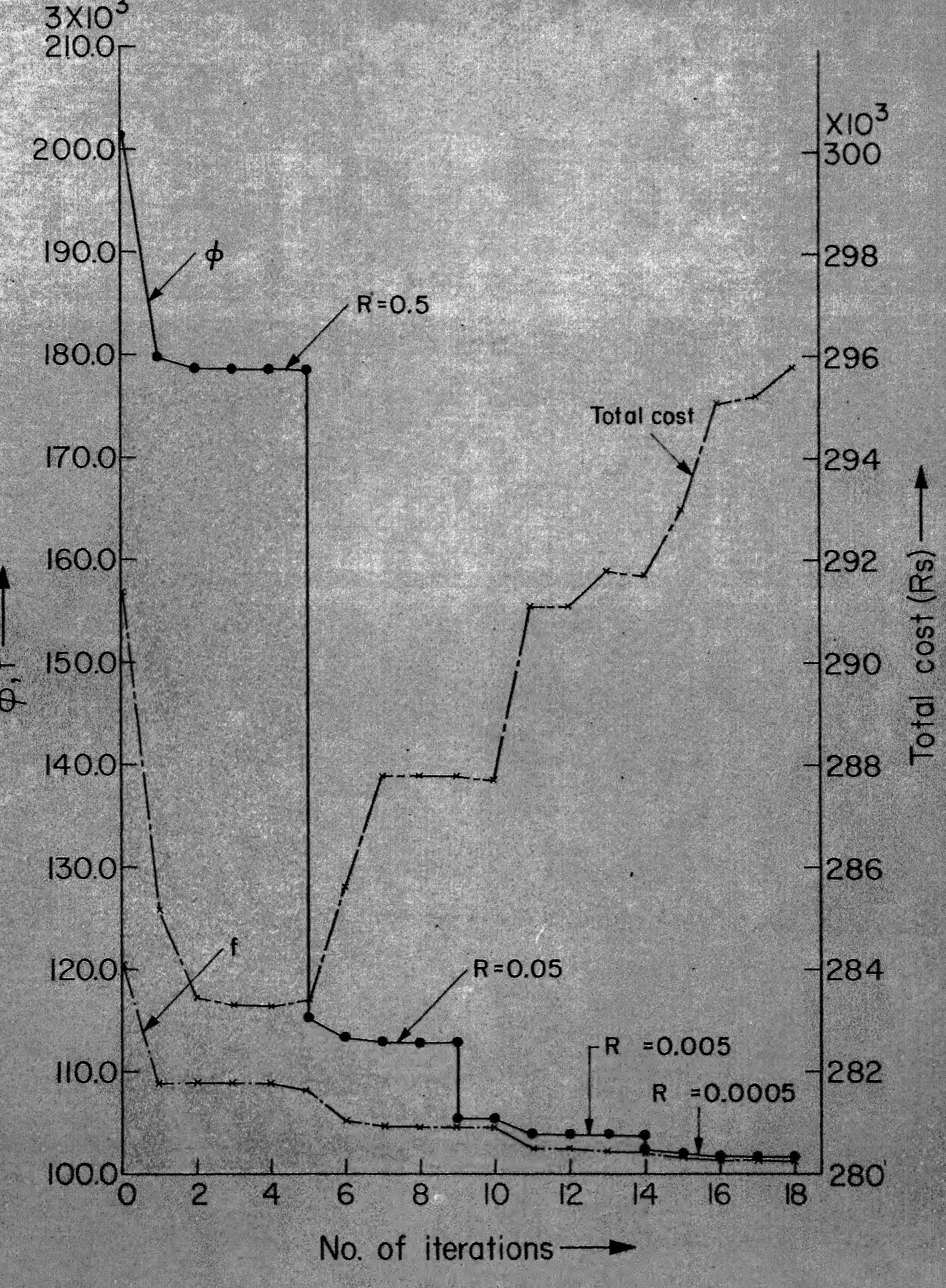


Fig.5.1 Progress of optimization procedure for problem 1

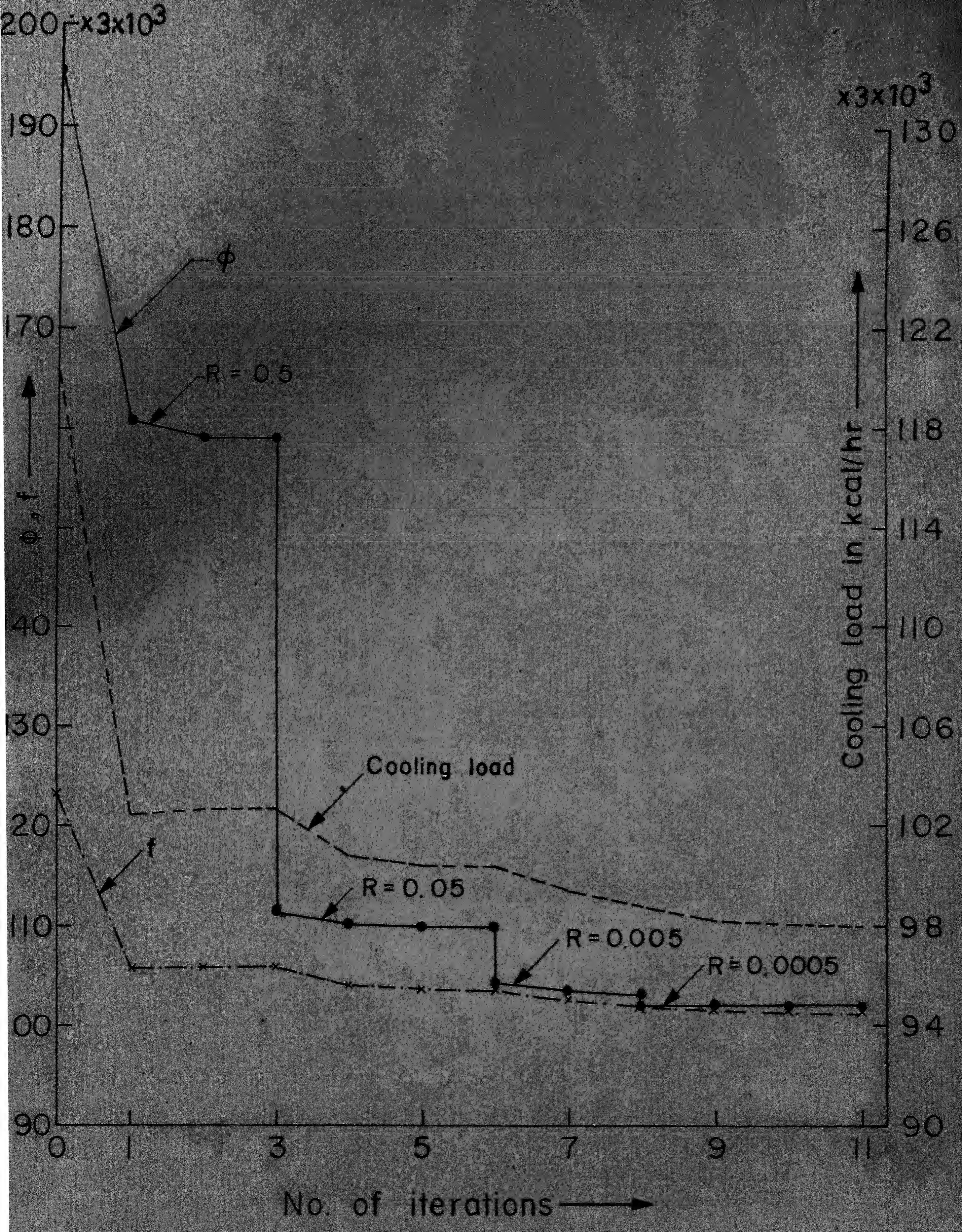


Fig.5.2 Progress of optimization procedure for problem 2

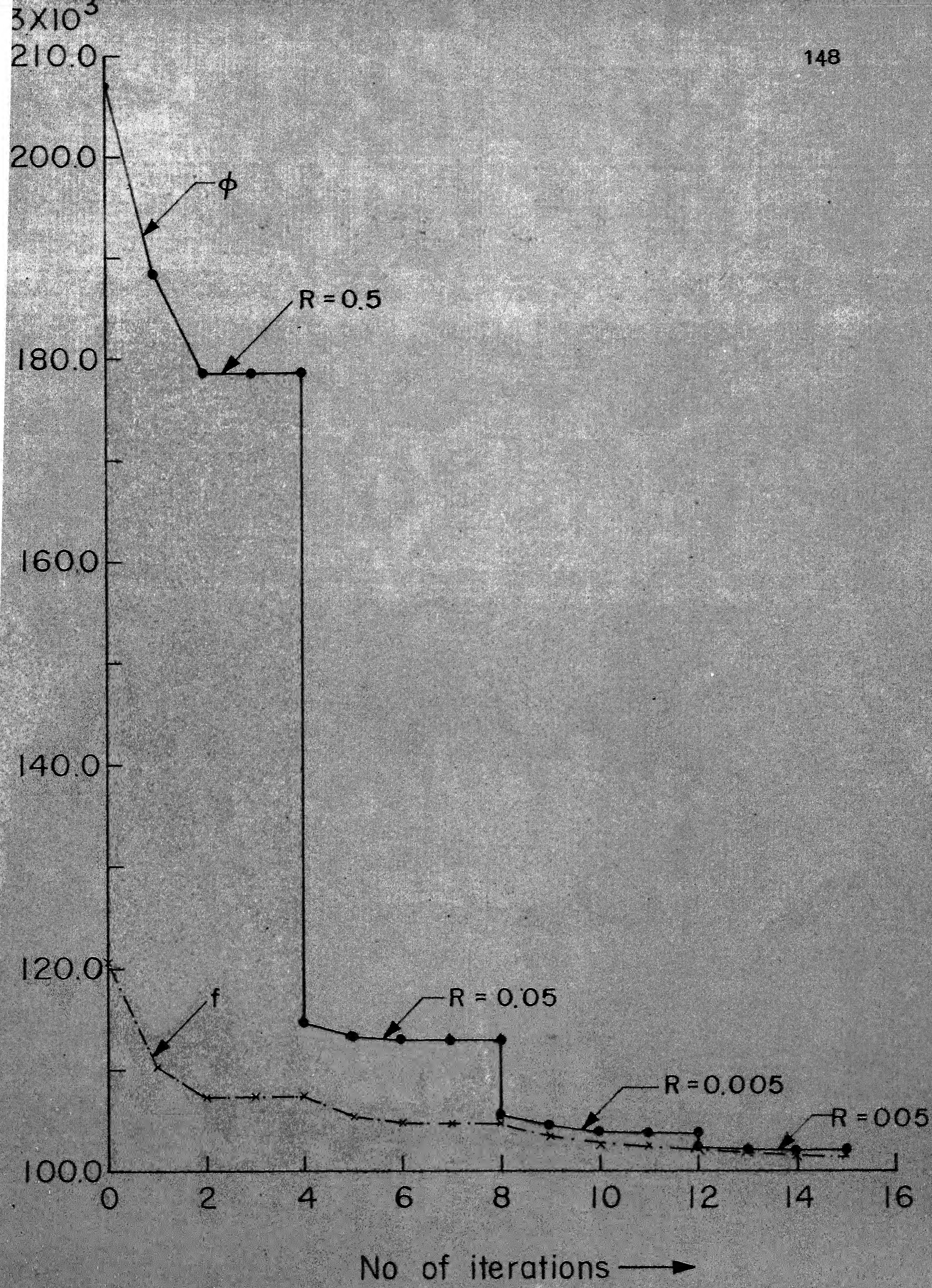


Fig.5.3 Progress of optimization procedure for problem 3

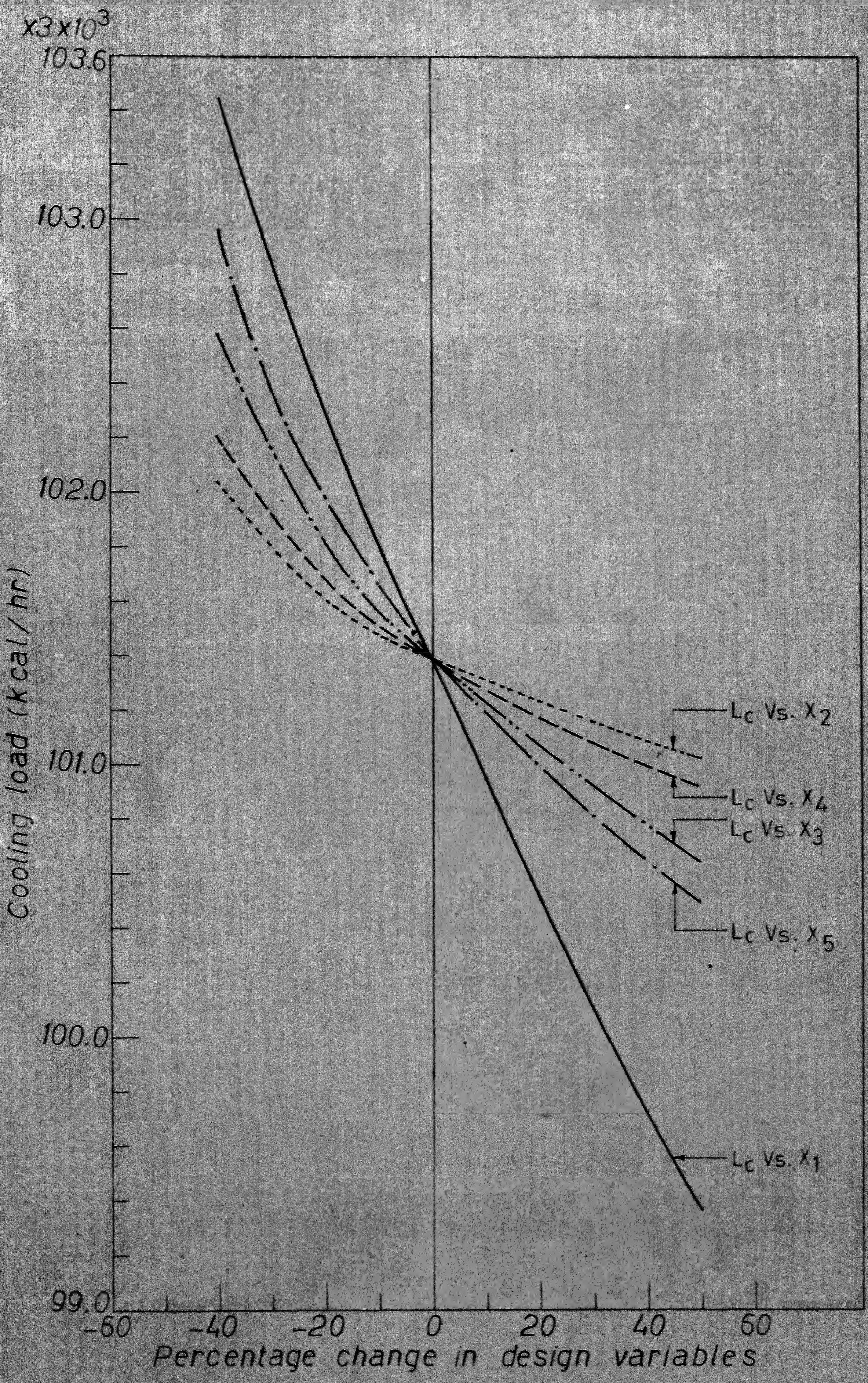


FIG. 5.4 SENSITIVITY ANALYSIS OF COOLING LOAD.

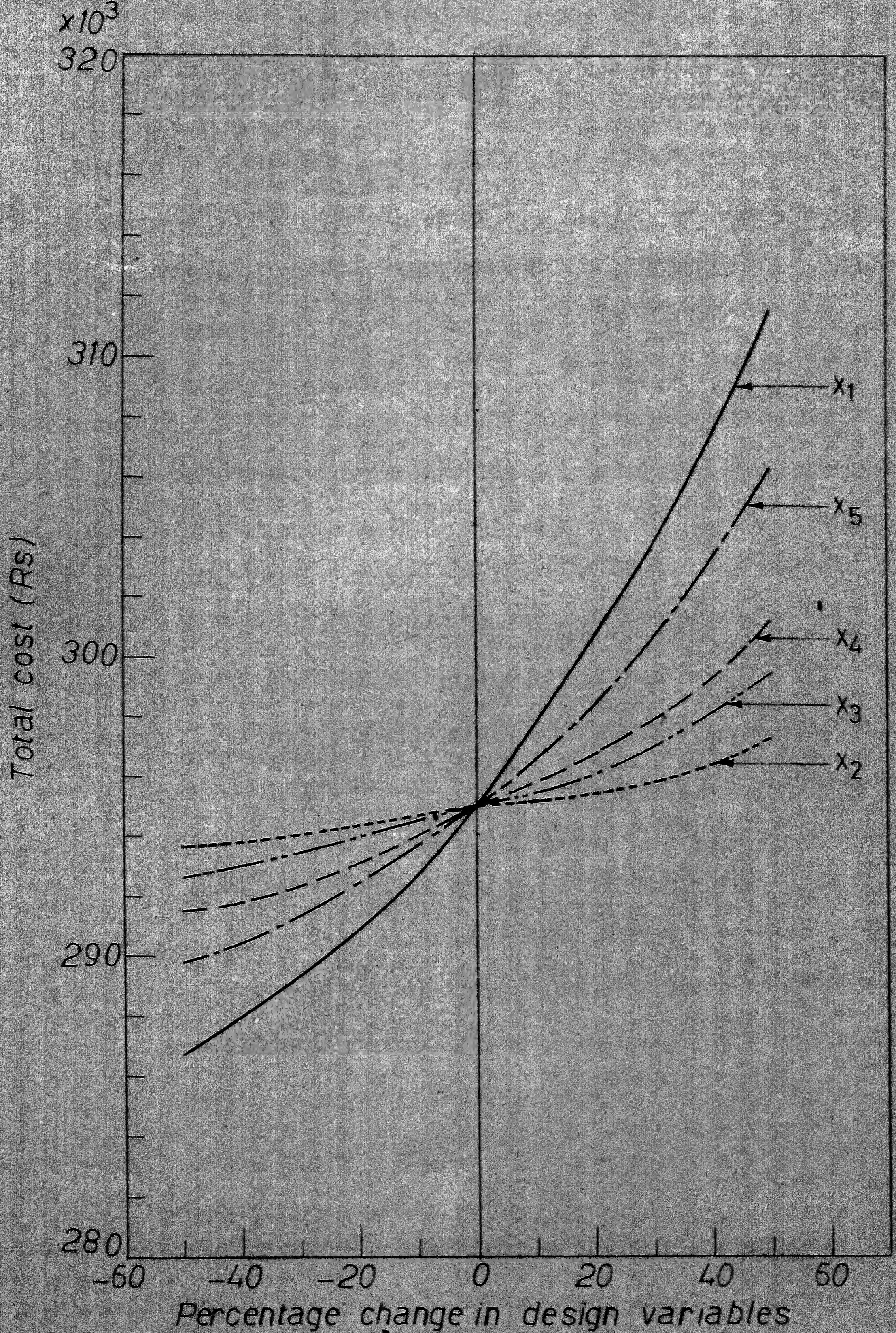


FIG.5.5 SENSITIVITY ANALYSIS OF TOTAL COST.

CHAPTER 6

APPLICATION OF EXTREMAL DISTRIBUTIONS

In most of the engineering design problems, design variables are random in nature. In many situations, concern often lies with the largest or smallest of a number of random variables. Success or failure of a system may rest solely on its ability to function under the maximum demand (load) or minimum capacity (strength), not simply the typical values. Floods, winds, temperatures, solar radiations and floor loadings are all variables whose largest value in a sequence may be critical to several engineering systems. Similarly the smallest values of strength of materials will also be critical in some situations. A systematic study of the maximum or minimum values of the random variables influencing the design may lead to a better design. Further the concept of finite life design can also be incorporated and thus a better use of materials can be made and technological obsolescenece can be cared for. A basic argument against the study of these extreme values is that as their probabilities are small, the error involved in their computation may be very large. However it has been found by Gumbel [24] that extreme values are much more reliable than the median whose value increases with simple size. Although extremal distributions have been applied

to several engineering disciplines in the past, no application has been reported in the field of thermal systems design. The application of extremal distributions in the design of heating and cooling systems is considered in this chapter.

In the present work, the hourly data is used to fit three types of extremal (asymptotic) distributions for temperature and solar radiation. Since type III distribution for the largest values has been found to fit the daily and yearly maximum values of dry bulb temperature and daily maximum values of solar radiation very well, it has been used in the optimization work. The optimum design of a refrigerated warehouse is considered with the mean value of the cooling load as the objective and an upper bound on the probability of occurrence of a specified extreme (permissible) value of temperature inside the warehouse. Upper and lower bounds on the design variables as well as an upper bound on the standard deviation of the cooling load are also considered in the formulation of this problems.

6.1 Extremal Distributions

Let X be a random variable and n tests be performed to find the probability characteristics of X. Let the results of the test (values realized for X) be arranged in increasing order of magnitude X_1', X_2', ..., so that the smallest value realized for X be X_1' and the largest X_n'. Let these n tests be

repeated several times and the results X_1', X_1'', X_1''', ..., X_2', X_2'', X_2''', ..., X_n', X_n'', X_n''', ... be fitted into a distribution $F_X(x)$. Once the distribution of X is known, the distributions of the smallest value X_1 and the largest value X_n can also be obtained. The essential conditions are: (i) Statistical variates are dealt with (ii) The parent distribution $[F_X(x)]$ from which the extreme values have been drawn and its parameters remain constant from one sample to next (iii) The observed extremes should be extremes of samples of independent data.

Distribution of the smallest value X_1:

$$P(X_1 > x) = P(\text{all n of the } X_i > x)$$

If X_i are independent,

$$\begin{aligned} P(X_1 > x) &= P(X_1 > x)\, P(X_2 > x) \ldots P(X_n > x) \\ &= [1 - F_{X_1}(x)][1 - F_{X_2}(x)] \ldots [1 - F_{X_n}(x)] \end{aligned}$$

In the specified case where all the X_i are identically distributed with cumulative distribution function $F_X(x)$,

$$P(X_1 > x) = [1 - F_X(x)]^n \tag{6.1}$$

$$F_{X_1}(x) = P(X_1 \leq x) = 1 - [1 - F_X(x)]^n \tag{6.2}$$

Distribution of largest value X_n:

As in the case of the smallest value X_1, if X_i are assumed to be independent and identically distributed,

$$P(X_n \leq x) = F_{X_n}(x) = [F_X(x)]^n \tag{6.3}$$

Knowing from past experience the distribution of $\underline{X}_i$ (say, temperature) in any (i^{th}) year, one might need the distribution of, say, $\underline{X}_{50}$, the largest temperature in 50 years, this being the design life time of a proposed thermal system. Equations (6.2) and (6.3) represent the exact probability distributions of the extreme values in terms of the parent distribution function, $F_X(x)$.

If the conditions of independence and common distribution hold among the X_i, then in most of the practical cases the shape of the distribution of extreme values (X_1 or X_n) is relatively insensitive to the exact shape of the distribution of X_i. In such cases limiting forms (as n grows) of the distributions of X_1 and X_n can be found, which can be expected to describe the behaviour of that random variable even when the exact shape of the distribution of X is not known precisely. These limiting distributions are called asymptotic distributions. The theory of asymptotic distributions may also be applied to many practical situations even when the assumption of independence is not strictly satisfied provided sufficiently large samples are used for obtaining the parameters of the extremal (asymptotic) distributions.

Following are the specific distributions which are applied widely in engineering applications.

6.1.1 Type I distribution for largest value

If the parent distribution is of the type

$$F_X(x) = [1 - e^{-g(x)}] \qquad (6.4)$$

where $g(x)$ is an increasing function of x, then the distribution function of the largest value, $F_Y(y)$, is given by

$$F_Y(y) \equiv F_{X_n}(y) = \exp\{-\exp[-\alpha_1(y-u)]\}, \quad -\infty \leq y \leq \infty \qquad (6.5)$$

and the probability density function by

$$f_Y(y) = \alpha_1 \exp[-\alpha_1(y-u) - e^{-\alpha_1(y-u)}] \qquad (6.6)$$

The parameters α_1 and u in Eqs. (6.5) and (6.6) are to be estimated from the observed data.

6.1.2 Type I distribution for smallest value

The probability distribution and density functions for the smallest value are given as follows

$$F_Z(z) \equiv F_{X_1}(z) = 1 - \exp\{-\exp[-\alpha_1(z-u)]\}, \quad -\infty \leq z \leq \infty \qquad (6.7)$$

and

$$f_Z(z) = \alpha_1 \exp\{\alpha_1(z-u) - \exp[\alpha_1(z-u)]\} \qquad (6.8)$$

6.1.3 Type II distribution for largest value

If the parent distribution is

$$F_X(x) = 1 - \beta\left(\frac{1}{x}\right)^{K_1}, \qquad x \geq 0 . \tag{6.9}$$

then the probability distribution and density functions of the largest value (Y) are given as

$$F_Y(y) = F_{X_n}(y) = \exp\left[-\left(\frac{u}{y}\right)^{K_1}\right] \tag{6.10}$$

and

$$f_Y(y) = \frac{K_1}{u}\left(\frac{u}{y}\right)^{K_1+1} \exp\left[-\left(\frac{u}{y}\right)^{K_1}\right] \quad y \geq 0 \tag{6.11}$$

where the parameters K_1 and u are to be determined from the observed data.

6.1.4 Type II distribution for smallest value

The probability distribution and density functions for the smallest value are given as follows

$$F_Z(z) = 1 - \exp\left[-\left(\frac{u}{z}\right)^{K_1}\right] \tag{6.12}$$

and

$$f_Z(z) = -\frac{K_1}{u}\left(\frac{u}{z}\right)^{K_1+1} \exp\left[-\left(\frac{u}{z}\right)^{K_1}\right] \tag{6.13}$$

6.1.5 Type III distribution for largest value

If the parent distribution is of the form

$$F_X(x) = 1 - c(\omega_1 - x)^{K_1}, \quad x \leq \omega_1 ; \quad K_1 > 0 \tag{6.14}$$

then the probability distribution and density functions are given as

$$F_Y(y) = \exp\left[-\left(\frac{\omega_1 - y}{\omega_1 - u}\right)^{K_1}\right], \qquad y \leq \omega_1 \tag{6.15}$$

and

$$f_Y(y) = \frac{K_1}{\omega_1 - u} \left(\frac{\omega_1 - y}{\omega_1 - u}\right)^{K_1-1} \exp\left\{-\left(\frac{\omega_1 - y}{\omega_1 - u}\right)^{K_1}\right\},$$

$$y \leq \omega_1 \qquad (6.16)$$

where the parameters K_1 and u are to be determined from observed data.

6.1.6 Type III distribution for smallest value

If

$$F_X(x) = c(x - \varepsilon)^{K_1}, \qquad (6.17)$$

then

$$F_Z(z) = F_{X_1}(z) = 1 - \exp\left\{-\left(\frac{z - \varepsilon}{u - \varepsilon}\right)^{K_1}\right\}, \quad z \leq \varepsilon \qquad (6.18)$$

and

$$f_Z(z) = \frac{K_1}{u - \varepsilon} \left(\frac{z - \varepsilon}{u - \varepsilon}\right)^{K_1-1} \exp\left\{-\left(\frac{z - \varepsilon}{u - \varepsilon}\right)^{K_1}\right\}, \quad z \leq \varepsilon \qquad (6.19)$$

where the parameters u, ε and K_1 are determined from the observed data.

These three types of distributions for largest and smallest values are used in different displines of engineering. Type I distributions are applied to describe strength of brittle materials and hydrological phenomena, type II distributions are used to represent annual maximum wind and several meteorological and hydrological phenomena while type III distribution is used to study strength of metals in tension and fatigue.

6.2 Fitting Extremal Distributions for Meteorological Data

The above-mentioned distributions are fitted into maximum daily temperature and solar radiations for a year for two cities of India. They are also tried with yearly maximum temperature. The unknown parameters of the distributions are determined from the observed values. For fitting the data, the expressions (6.5), (6.10) and (6.15) are rewritten as follows.

The distribution function for type I (for largest value) is given by Eq. (6.5) as

$$F_Y(y) = \exp\{-\exp[-\alpha_1(y-u)]\} \tag{6.20}$$

Taking logarithm of Eq. (6.20), we obtain

$$-\ln F_Y(y) = \exp[-\alpha_1(y-u)] \tag{6.21}$$

i.e.

$$\ln\left(\frac{1}{F_Y(y)}\right) = \exp[-\alpha_1(y-u)] \tag{6.22}$$

By taking logarithms once again, we get

$$\frac{1}{\alpha_1}\ln\ln\left(\frac{1}{F_Y(y)}\right) = -y+u \tag{6.23}$$

i.e.

$$y = -\frac{1}{\alpha_1}\ln\ln\left(\frac{1}{F_Y(y)}\right) + u \tag{6.24}$$

Eq. (6.24) represents a straight line if y is plotted as ordinate against $-\ln\ln\left(\frac{1}{F_Y(y)}\right)$ as abscissa, provided the parent distribution given by Eq. (6.4) holds true for the phenomenon.

Similarly the distribution function for type II (for largest value) is given by Eq. (6.10) as

$$F_Y(y) = \exp\left[-\left(\frac{u}{y}\right)^{K_1}\right] \tag{6.25}$$

By taking logarithms of Eq. (6.25) twice and rearranging the terms, we obtain

$$\ln Y = -\frac{1}{K_1}\ln\ln\left(\frac{1}{F_Y(y)}\right) + \ln u \tag{6.26}$$

Thus if ln y is plotted as ordinate against $-\ln\ln\left(\frac{1}{F_y(y)}\right)$ as abscissa, then Eq. (6.26) can be represented as a straight line provided the parent distribution given by Eq. (6.10) holds for the random variable.

Lastly the distribution function for type III (for largest value) as represented by Eq. (6.15) is

$$F_Y(y) = \exp\left[-\left(\frac{\omega_1 - y}{\omega_1 - u}\right)^{K_1}\right] \tag{6.27}$$

where $y \leq \omega_1$ and $K_1 > 0$

Again taking logarithm of Eq. (6.27) and rearranging the terms, the following expression is obtained.

$$\ln(\omega_1 - y) = \frac{1}{K_1}\ln\ln\left(\frac{1}{F_Y(y)}\right) + \ln(\omega_1 - u) \tag{6.28}$$

Eq. (6.28) with $\ln(\omega_1 - y)$ as ordinate and $\ln\ln\left(\frac{1}{F_Y(y)}\right)$ as abscissa can be represented as a straight line if the parent distribution given by Eq. (6.15) holds true.

From the available hourly temperature and global solar radiation data, first the maximum of daily temperature and global solar radiation are selected. The maximum values so obtained are used to determine the values of abscissa and ordinate at each point for the three types of distributions. For this the N individual observations $(x_1, x_2, \ldots, x_m, \ldots, x_N)$ are arranged in increasing (decreasing) order of magnitude for maxima (minima). Then the N fractions $\frac{m}{(N+1)}$, $m = 1, 2, \ldots, N$, are calculated and the points $[\{x_m, \frac{m}{(N+1)}\}]$ are plotted on extremal probability paper. Once the different points are plotted, then a straight line is fitted through these points using least squares method. The error can also be computed between the exact point and the corresponding point on the straight line. The magnitude of this error is a gauge to measure which of the distributions fits best in the observed data. In Eq. (6.28), ω_1 was assumed to be little higher than the observed maximum-most value. The results obtained are given in Tables (6.1) and (6.2) and are shown graphically in Figures 6.1 to 6.6 for the dry bulb temperature and global solar radiation of New Delhi and Roorkee, India. These results indicate that type III distribution fits best for temperature as well as solar radiation data.

To find the difference between extremal distribution and the commonly used normal distribution, the daily maximum values of temperature are fitted into normal distribution and the resulting density function is compared with the type III extremal density function given by Eq. (6.16) in Figure 6.7.

6.3 Application to Design of Thermal Systems

To demonstrate the use of extremal distributions in the design of thermal systems, the optimum design of a refrigerated warehouse is considered. For this the type III extremal distribution based on yearly maximum values of dry bulb temperature for 25 years (for Poona, India) is used. All the three types of distribution are fitted to this data using Eqs. (6.24), (6.26) and (6.28), and the results are shown in Table 6.3 and Figures 6.8 to 6.11. It is found that type III distribution for the largest value fits the data best. Then the hourly values of temperature are computed using this maximum value. The temperature range and the percentage daily range are taken as the average of the values of twelve days of the year, one selected from each month on which the temperature is maximum (in that month).

The hourly temperature is calculated as

$$t_h = t_{max} - DLY * PDLY_h \qquad (6.29)$$

where t_h = temperature at any hour h,

DLY = daily range of temperature, and

$PDLY_h$ = percentage daily range at h^{th} hour of the day.

The hourly values of solar radiations are taken over the day having the maximum value of mean plus three times the standard deviation. In this design procedure only one such critical day is taken as the design day.

6.4 Problem Formulation and Numerical Results

Here the aim is to find insulation thicknesses for the walls and roof and maximum design temperature for (i) minimum mean cooling load, (ii) minimum total cost and (iii) minimum of the weighted sum of total cost and cooling load. Further, upper and lower bounds are prescribed on the insulation thicknesses, the probability of occurrence of the maximum design temperature is restricted to be less than a prescribed value and the standard deviation of the load is constrained to be less than a certain fraction of its mean value. The problem with the first objective can be stated as a standard optimization problem as:

$$\text{Find} \quad \vec{X} = \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \end{Bmatrix}$$

such that $F(\vec{X}) = \bar{L}_c$ minimum, subjected to

$$\begin{aligned}
g_1, g_2 &: \quad 0.005 \le X_1 \le 0.15 \\
g_3, g_4 &: \quad 0.005 \le X_2 \le 0.15 \\
g_5, g_6 &: \quad 0.005 \le X_3 \le 0.15 \\
g_7, g_8 &: \quad 0.005 \le X_4 \le 0.15 \\
g_9, g_{10} &: \quad 0.005 \le X_5 \le 0.15
\end{aligned}$$

$$g_{11}: \quad K_3 - F_Y(x_6) \leq 0^{\oplus}$$

$$g_{12}: \quad x_6 - \omega_1 \leq 0$$

$$g_{13}: \quad \sigma_{L_c} - K_2\bar{L}_c \leq 0$$

$$g_{14}: \quad 40 - x_6 \leq 0$$

where X_1 is the thickness of roof insulation, X_2, X_3, X_4 and X_5 are insulation thicknesses of walls and X_6 is the maximum temperature, $F_Y(x_6)$ is the probability of occurrence of temperature less than x_6, ω_1 is the maximum possible value of x_6, σ_{L_c} is the standard deviation of the cooling load, $\bar{L}_c$ is the mean value of the cooling load and K_2, K_3 are constants.

The value of the probability $F_Y(x_6)$ is calculated using Eq. (6.15) with the values of constants evaluated from the equation of straight line (fitted by least squares method) as follows.

The equation of the straight line obtained is

$$\ln(\omega_1 - y) = 0.2992861 \ln \ln\left(\frac{1}{F_Y(y)}\right) + 1.231853 \qquad (6.31)$$

By matching the coefficients of Eq. (6.31) with those of Eq. (6.28), we obtain

$$\frac{1}{K_1} = 0.2992861 \quad \text{or} \quad K_1 = 3.3401 \qquad (6.32)$$

$$\text{and} \quad \ln(\omega_1 - u) = 1.231853 \quad \text{or} \quad (\omega_1 - u) = 3.4212 \qquad (6.33)$$

$\oplus$ x_6 represents the particular value taken by X_6.

The value of the constant K_3 in the constraint $K_3 - F_Y(x_6) \leq 0$ is taken as 0.05. The three optimization problems stated above are solved using the procedure described in Chapter 2 and the results are given in Table 6.4. The progress of optimization for the three problems is shown in Figures 6.11 to 6.13.

The minimum cooling load obtained in case of the first problem is 3,19,613.7 kcal/hr while it was 3,03,854.7 kcal/hr in the case of probabilistic design (Chapter 5). Thus there is an increase of 5.08% in the cooling load due to a change in the design criterion. The three problems solved to illustrate the application of extremal distributions give different cooling loads and annual costs. The reduction in cooling load is 18.6% in problem 1 while it is 12.7% and 17.5% in the second and third problems respectively. Similarly, the cost reductions in the three problems are 2.7, 5.2 and 3.6% respectively. It is interesting to note that in the first problem, the percentage load reduction is maximum while that of cost is minimum compared to the other two problems. In problem three, where equal weightages are given to cooling load and cost, the amount of reduction obtained in the cooling load or cost lies in between the values obtained in the other two cases. This shows that an objective of the type used in problem three has to be used whenever both energy conservation and investment are equally important.

6.5 Alternate Formulation of Design Problems

The problems of design of thermal systems can also be formulated in alternate forms using the concept of return period in extremal distributions. The return period T(x) is defined as:

$$T(x) = \frac{1}{1 - F_X(x)}$$

where $F_X(x)$ is the distribution function of X. Physically the return period indicates the number of observations during which, on the average, there is one observation equalling or exceeding x. Its value increases with that of the variate (x). It can be seen that to every distribution function there corresponds a return period and vice-versa. If the observations are made at constant intervals of time, the return period being a number of observations, is also a time, measured in the same units.

The return period corresponding to any extreme value can be obtained from the probability paper as shown in Figure 6.10 or corresponding to any return period the extreme value of the parameter can be obtained. Now the design problem can be formulated with this extreme value and the related return period as the life of the system. As an example, if the return period corresponding to the extreme temperature is to be obtained, a horizontal line FE is drawn from the desired value of maximum temperature, F (43.6°C) and from the point of intersection E of lines FE and AB (line fitted by the least

square error method for type III distribution), a vertical line is drawn until it intersects the abscissa at the line G. The value of the return period corresponding to this point is found to be 20 years in Figure 6.8. Since this temperature (43.6°C) is expected to be exceeded only after twenty years, the life of the system can be taken as this return period, namely, 20 years.

6.6 Kuhn-Tucker Conditions

The Kuhn-Tucker conditions are applied at the final design point $\vec{X}^*$ of problem 3 in which constraints 1 and 14 are active. For this problem the vector $\vec{E}$ and matrix [D] are found to be

$$\vec{E} = \begin{Bmatrix} 17.51 \\ 13.73 \\ 11.82 \\ 9.62 \\ 15.46 \\ 18.75 \end{Bmatrix}$$

and $$[D] = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

The value of the vector $\vec{\xi}$ is computed as

$$\vec{\xi} = \begin{Bmatrix} \xi_1 \\ \xi_{14} \end{Bmatrix} = \begin{Bmatrix} 17.51 \\ 18.75 \end{Bmatrix} > \vec{0}$$

Since both ξ_i are positive, Kuhn-Tucker conditions are satisfied and hence the final design vector $\vec{X}^*$ is guaranteed to be a relative minimum.

6.7 Sensitivity Analysis

Figures 6.14 and 6.15 show the influence of changing the design variables (from the optimum point) on the total cost and cooling load. It is seen that the cooling load as well as the total cost is most sensitive to the outside maximum design temperature. Among the insulation thicknesses, the roof thickness (X_1) has the maximum effect on the cooling load and total cost.

TABLE 6.1

PARAMETERS OF EXTREMAL DISTRIBUTIONS FOR NEW DELHI (INDIA)

(i) Dry bulb temperature

Type of distribution	Equation of straight line obtained by using least squares method	Magnitude of error (sum of squares of error for 365 points)	A	B
Type I	$A = 4.117271B + 26.8004$	187.708	y	$-\ln \ln(\frac{1}{F_Y(y)})$
Type II	$A = -0.167153B + 3.4813$	0.60792	$\ln y$	$-\ln \ln(\frac{1}{F_Y(y)})$
Type III	$A = 0.42630B + 2.8919$	0.362056	$\ln(\omega_1 - y)$	$\ln \ln(\frac{1}{F_Y(y)})$

(ii) Global solar radiations

Type of distribution	Equation of straight line obtained by using least squares method	Magnitude of error (sum of squares of error for 365 points)	A	B
Type I	$A = 12.99628B + 49.05519$	11709.3	y	$-\ln \ln(\frac{1}{F_Y(y)})$
Type II	$A = -0.3295B + 4.2968$	2.6735	$\ln y$	$-\ln \ln(\frac{1}{F_Y(y)})$
Type III	$A = 0.58676B + 3.3401$	0.98402	$\ln(\omega_1 - y)$	$\ln \ln (\frac{1}{F_Y(y)})$

TABLE 6.2

PARAMETERS OF EXTREMAL DISTRIBUTIONS FOR ROORKEE (INDIA)

(i) Dry bulb temperature

Type of distribution	Equation of straight line obtained by using least squares method	Magnitude of error (sum of squares of error for 365 points)	A	B
Type I	A = 4.84736B + 25.40185	39.973	y	$-\ln \ln(\frac{1}{F_Y(y)})$
Type II	A = -0.1868B + 3.4886	0.592133	$\ln y$	$-\ln \ln(\frac{1}{F_Y(y)})$
Type III	A = 0.43121B + 2.8635	0.21078	$\ln(\omega_1 - y)$	$\ln \ln(\frac{1}{F_Y(y)})$

(ii) Global solar radiations

Type of distribution	Equation of straight line obtained by using least squares method	Magnitude of error (sum of squares of error for 365 points)	A	B
Type I	A = 10.27853B + 31.74231	4713.14	y	$-\ln \ln(\frac{1}{F_Y(y)})$
Type II	A = -0.4584B - 3.8183	4.19108	$\ln y$	$-\ln \ln(\frac{1}{F_Y(y)})$
Type III	A = 0.4987B + 3.5713	0.93145	$\ln(\omega_1 - y)$	$\ln \ln(\frac{1}{F_Y(y)})$

TABLE 6.3

PARAMETERS OF EXTREMAL DISTRIBUTIONS FOR POONA (INDIA)

Yearly maximum dry bulb temperature

Type of distribution	Equation of straight line obtained by using least squares method	Magnitude of error (sum of squares of error for 25 points)	A	B
Type I	$A = 0.75890B + 41.4382$	0.8830	y	$-\ln \ln(\frac{1}{F_Y(y)})$
Type II	$A = -0.18713B + 3.7479$	0.209822	$\ln y$	$-\ln \ln(\frac{1}{F_Y(y)})$
Type III	$A = 0.29928B + 1.23185$	0.14848	$\ln(\omega_1 - y)$	$\ln \ln(\frac{1}{F_Y(y)})$

TABLE 6.4

RESULTS OF OPTIMIZATION

At initial design			At optimum design			% reduction in objective	Computer time	Cooling load at optimum point (kcal/hr)
Design variables	Penalty function	Objective function	Design variables	Penalty function	Objective function			
1. Minimization of cooling load as the objective of the problem								
X_1=0.05m			X_1=0.1483*					
X_2=0.05m			X_2=0.1431*				16	
X_3=0.05m	6,74,490.0	3,92,640.0 kcal/hr	X_3=0.1398	3,21,330.0	3,19,613.7 kcal/hr	18.7	minutes	3,19,613.7
X_4=0.05m			X_4=0.1427*				on IBM	
X_5=0.05m			X_5=0.1414*				370/155	
X_6=40.1°C			X_6=40.20				computer	
2. Minimization of total cost as the objective of the problem								
X_1=0.05m			X_1=0.1052m					
X_2=0.05m			X_2=0.0734m				40	
X_3=0.05m	4,89,629.8	Rs. 3,17,283.8	X_3=0.0694m	3,01,135.0	Rs. 3,00,943.6	5.2	minutes	3,42,804.0
X_4=0.05m			X_4=0.0812m				on IBM	
X_5=0.05m			X_5=0.0904m				7044	
X_6=40.1°C			X_6=40.002°C**				computer	

Continued...

Table 6.4 (Continued)

3. Minimization of weighted average of the cost and cooling load as the objective of the problem

At initial design			At optimum design			% reduction in objective	Computer time	Cooling load at optimum point (kcal/hr)
Design variables	Penalty function	Objective function	Design variables	Penalty function	Objective function			
X_1=0.05m			X_1=0.1411m*					
X_2=0.05m			X_2=0.1163m				40	
X_3=0.05m	9,79,259.8	6,34,567.6	X_3=0.1170m	5,68,199.0	5,67,504.0	10.6	minutes	3,23,448.0
X_4=0.05m			X_4=0.1281m				on IBM	
X_5=0.05m			X_5=0.1282m				7044	
X_6=40.1°C			X_6=40.002°C**				computer	

* Active constraints: Upper bound on insulation **thickness**

** Active constraints: Lower bound on the maximum temperature.

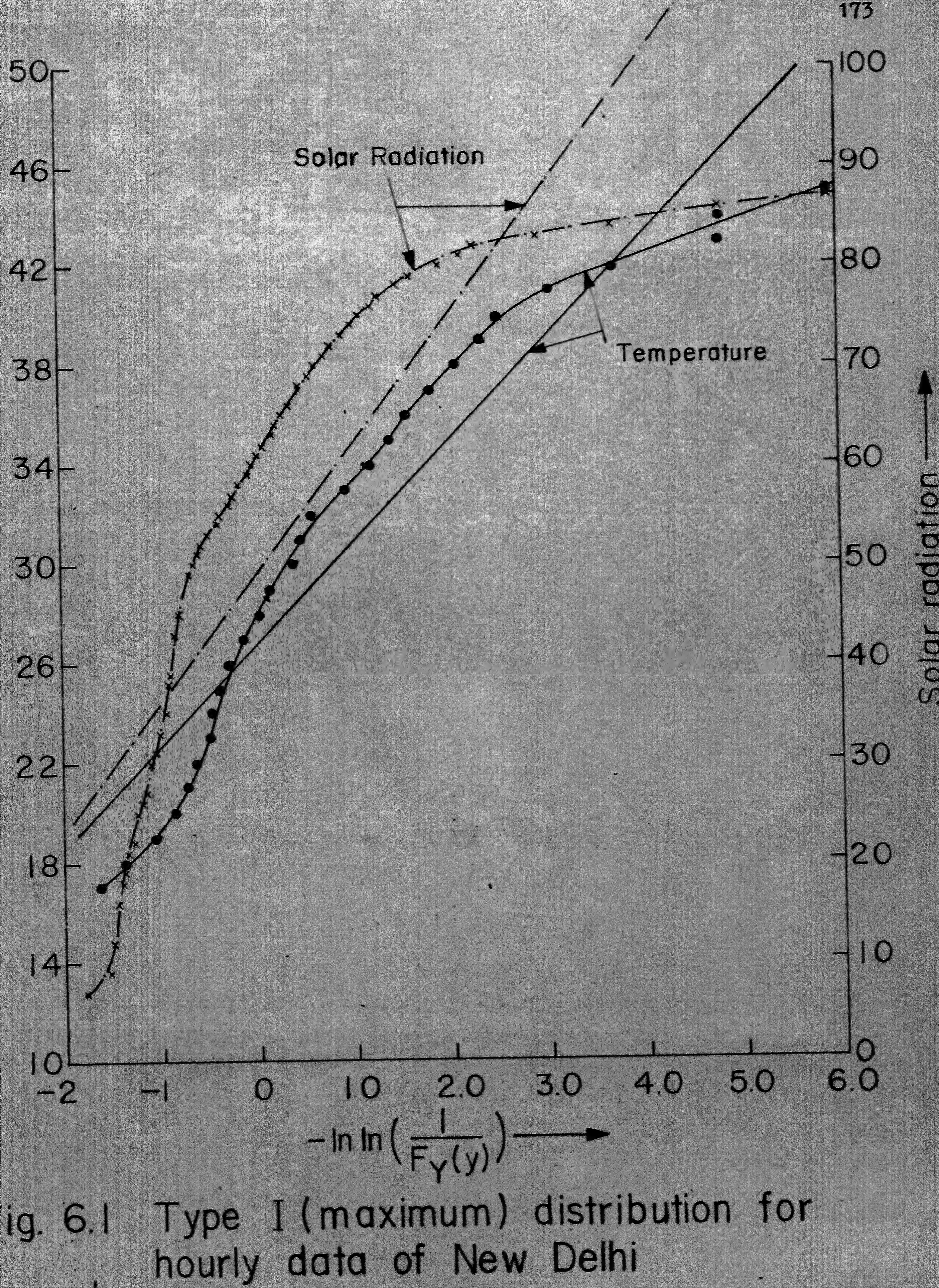


Fig. 6.1 Type I (maximum) distribution for hourly data of New Delhi

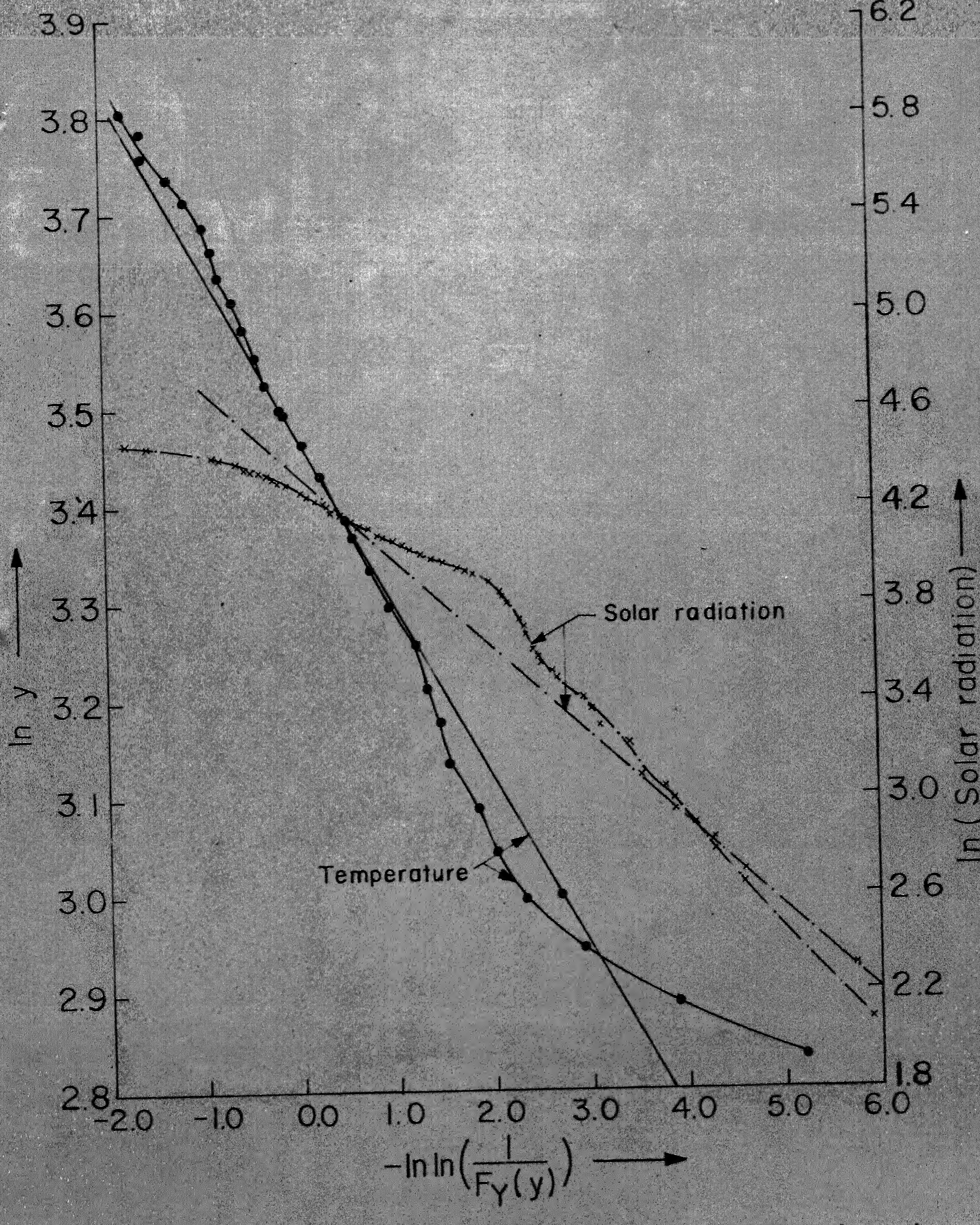


Fig.6.2 Type II (Maximum) distribution for hourly data of New Delhi

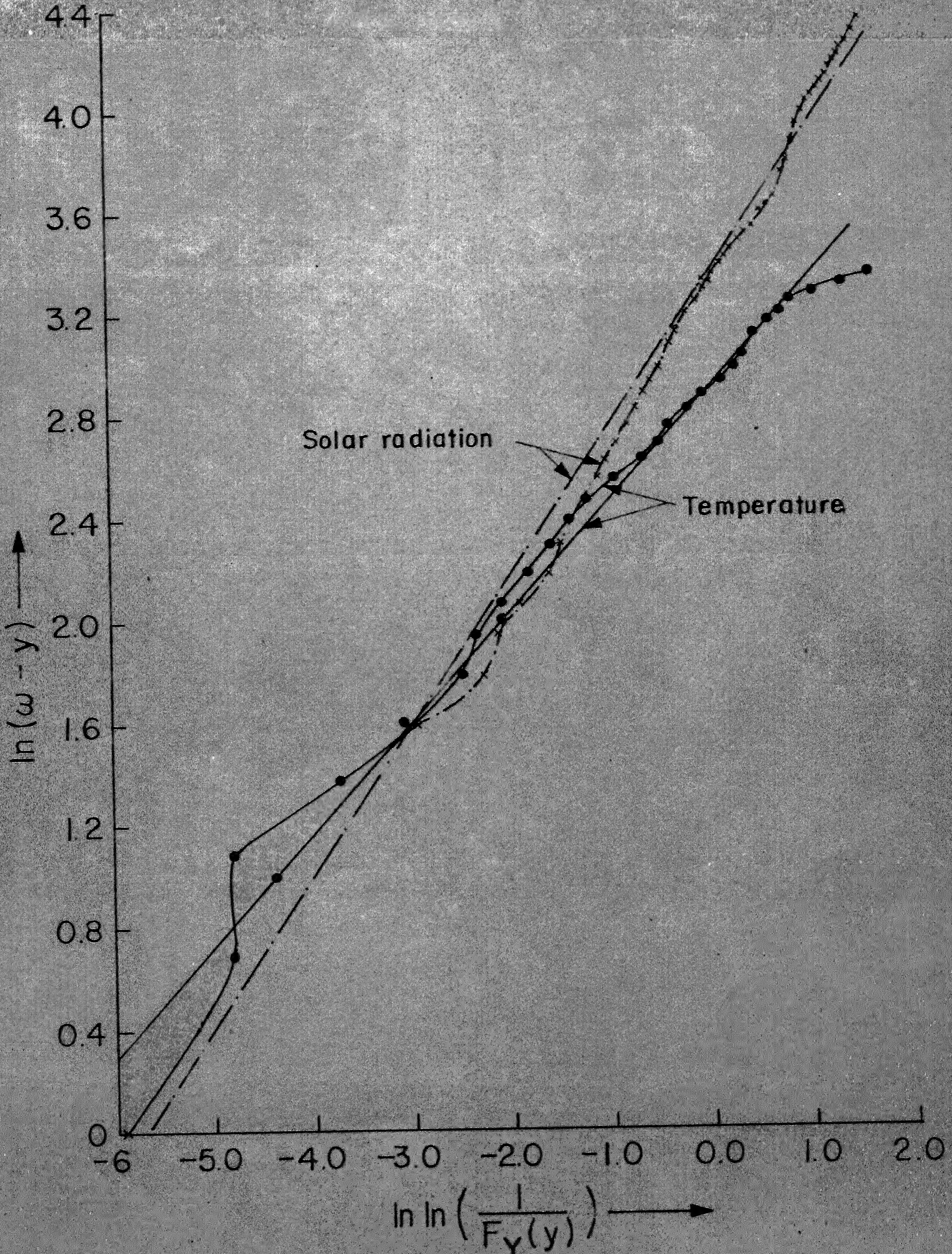


Fig. 6.3 Type III (maximum) distribution for hourly data of New Delhi

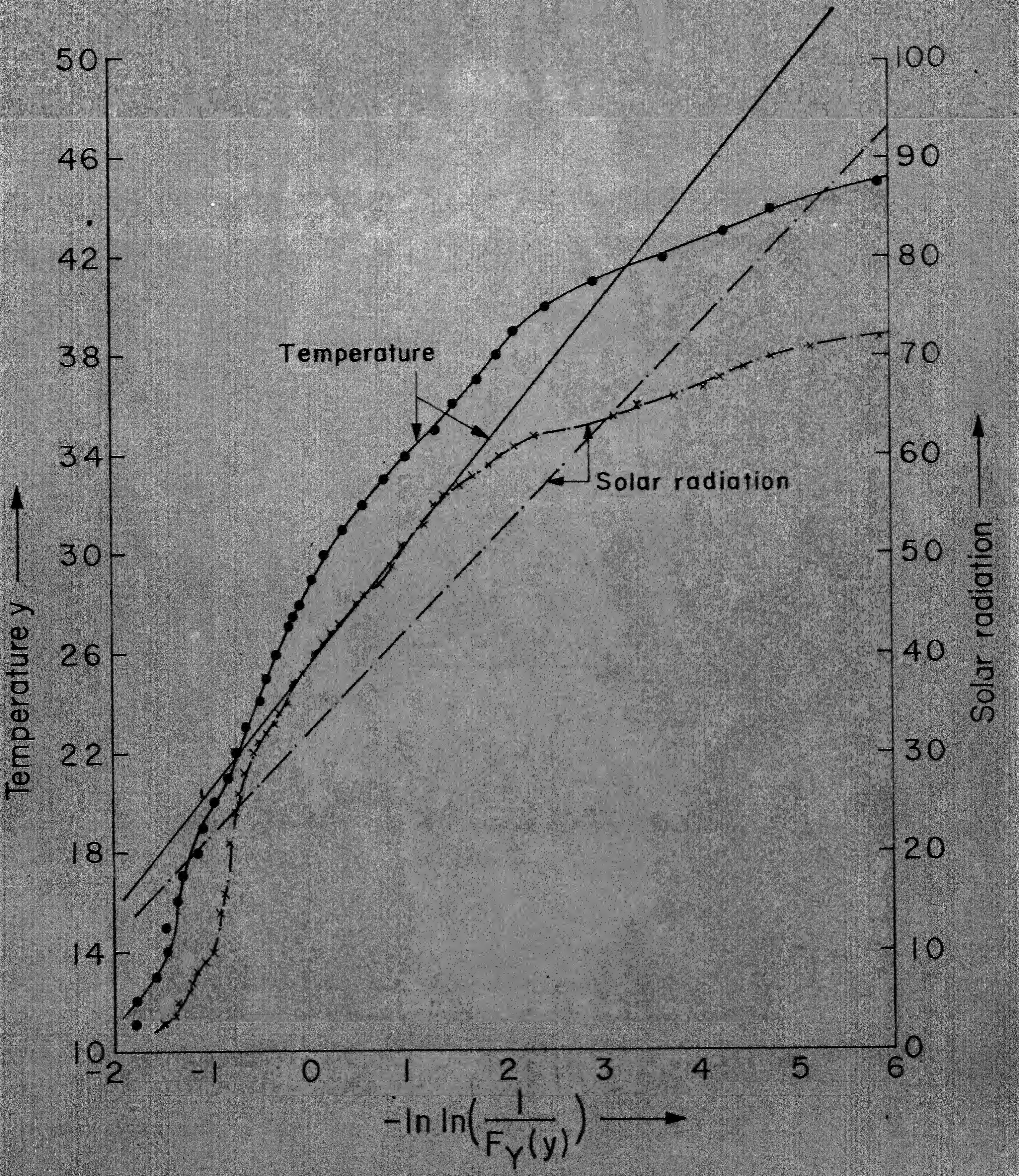


Fig. 6.4 Type I (maximum) distribution for hourly data of Roorkee

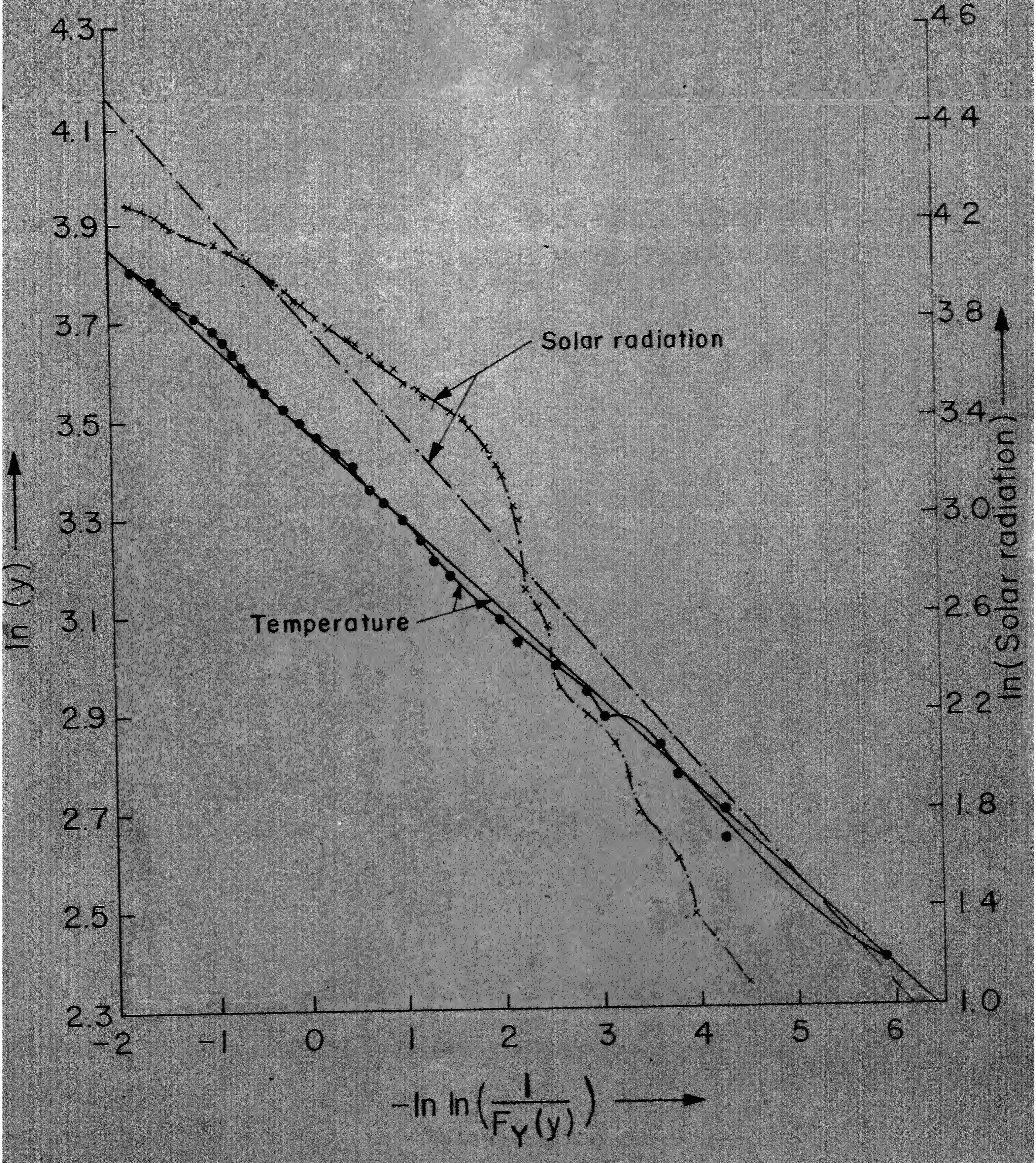


Fig. 6.5 Type II (maximum) distribution for hourly data of Roorkee

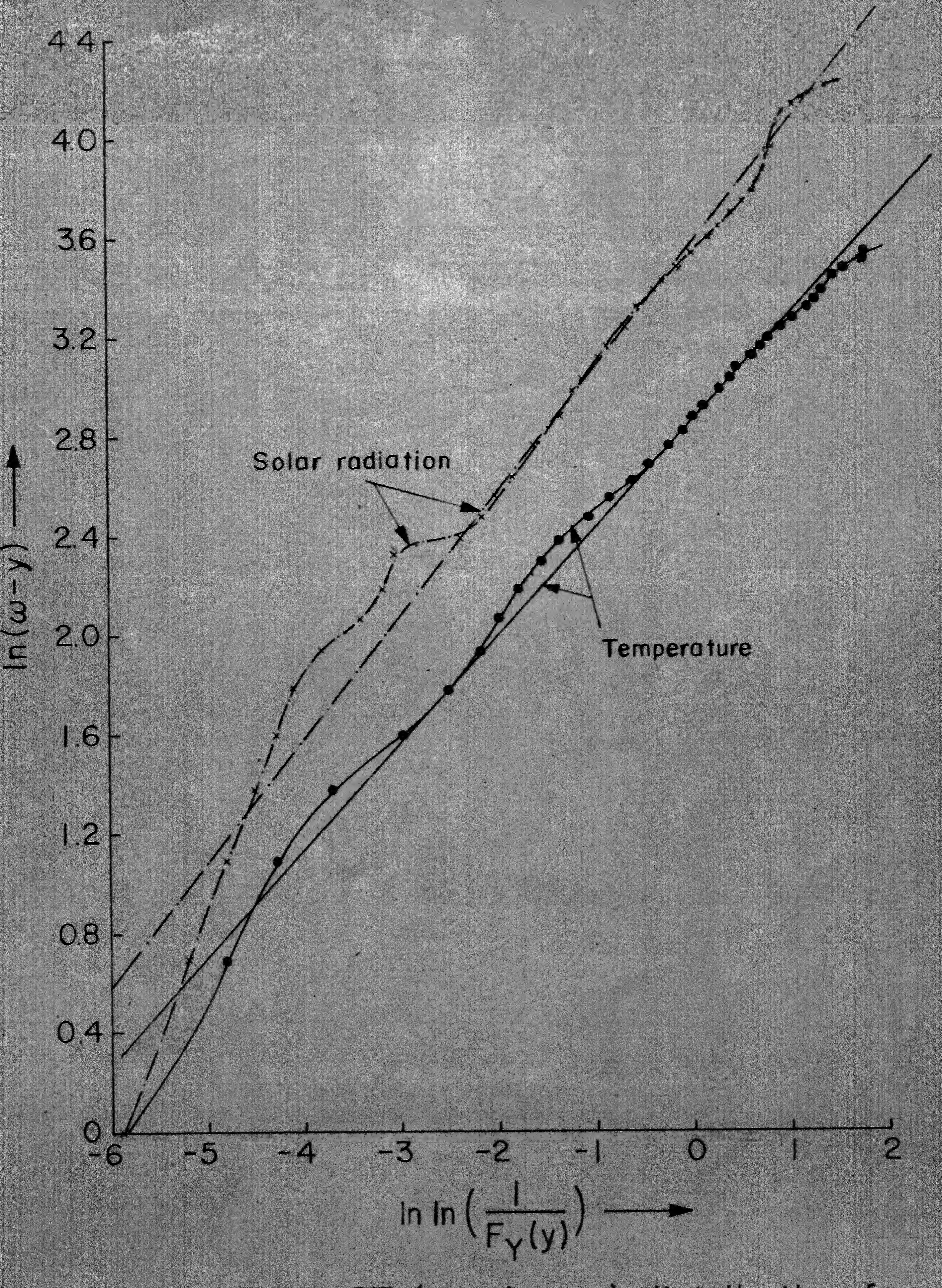


Fig. 6.6 Type III (maximum) distribution for hourly data of Roorkee

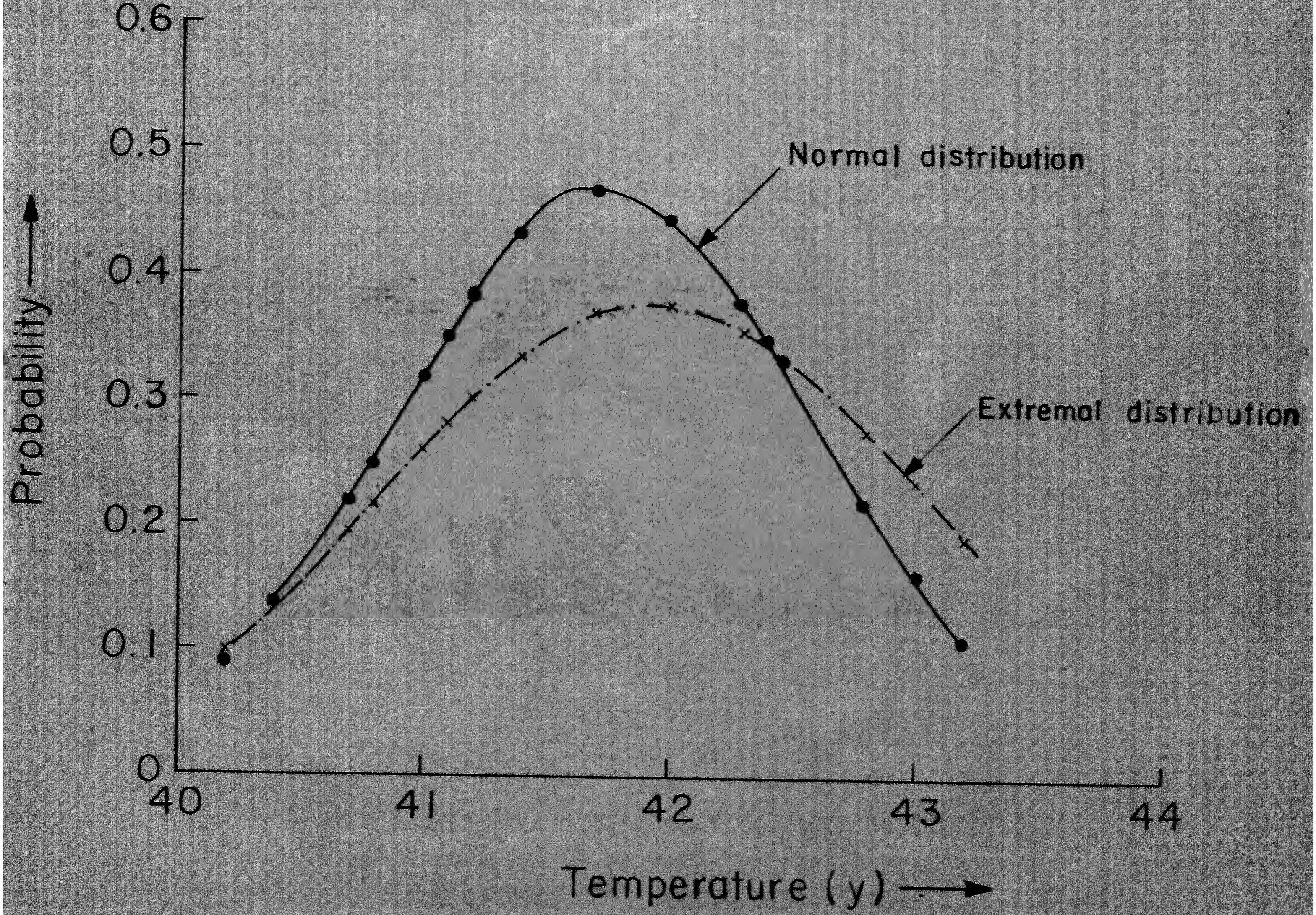


Fig. 6.7 Comparison of extremal type III and normal density functions corresponding to yearly maximum temperature for Poona

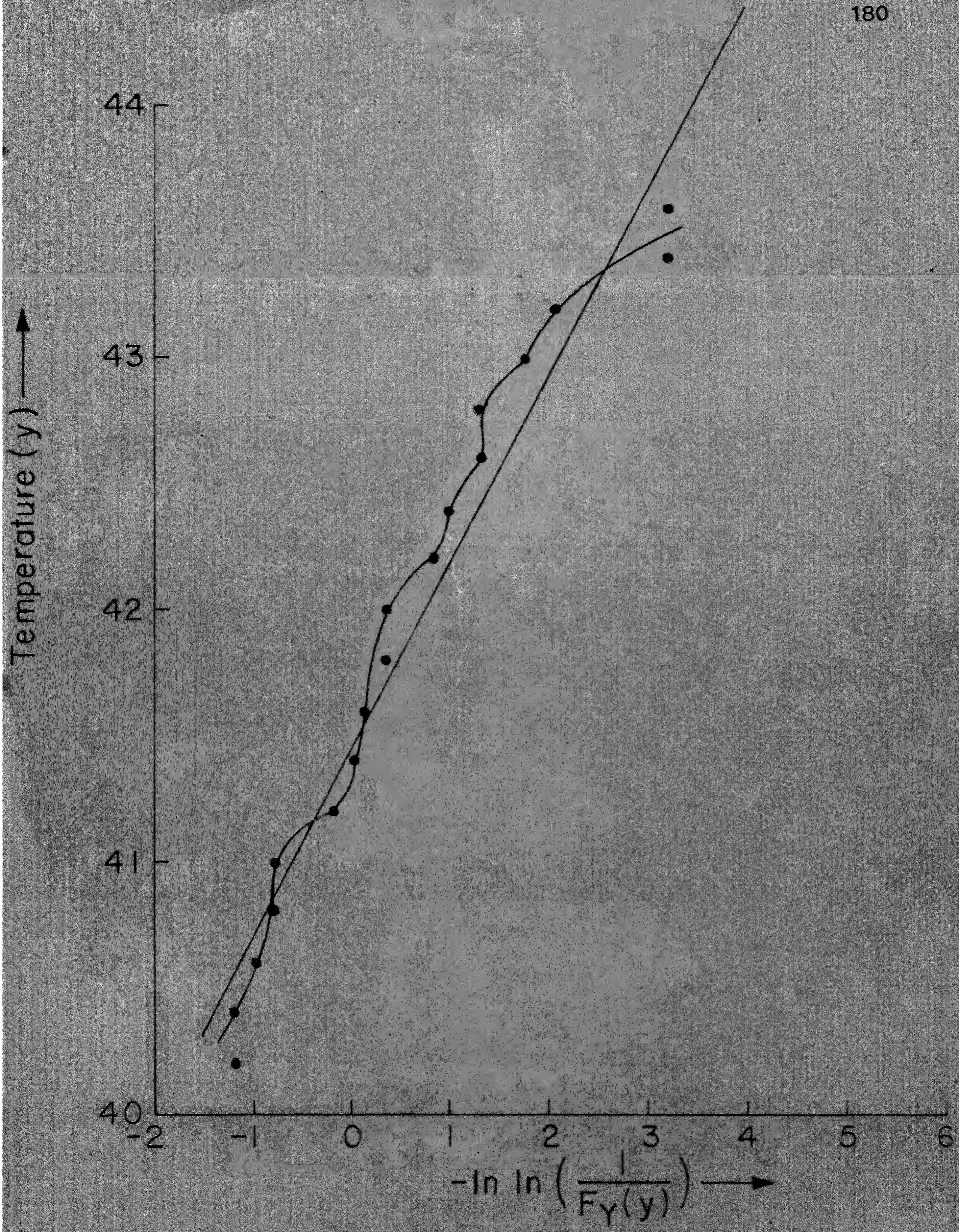


Fig. 6.8 Type I (maximum) distribution for yearly maximum temperature of Poona

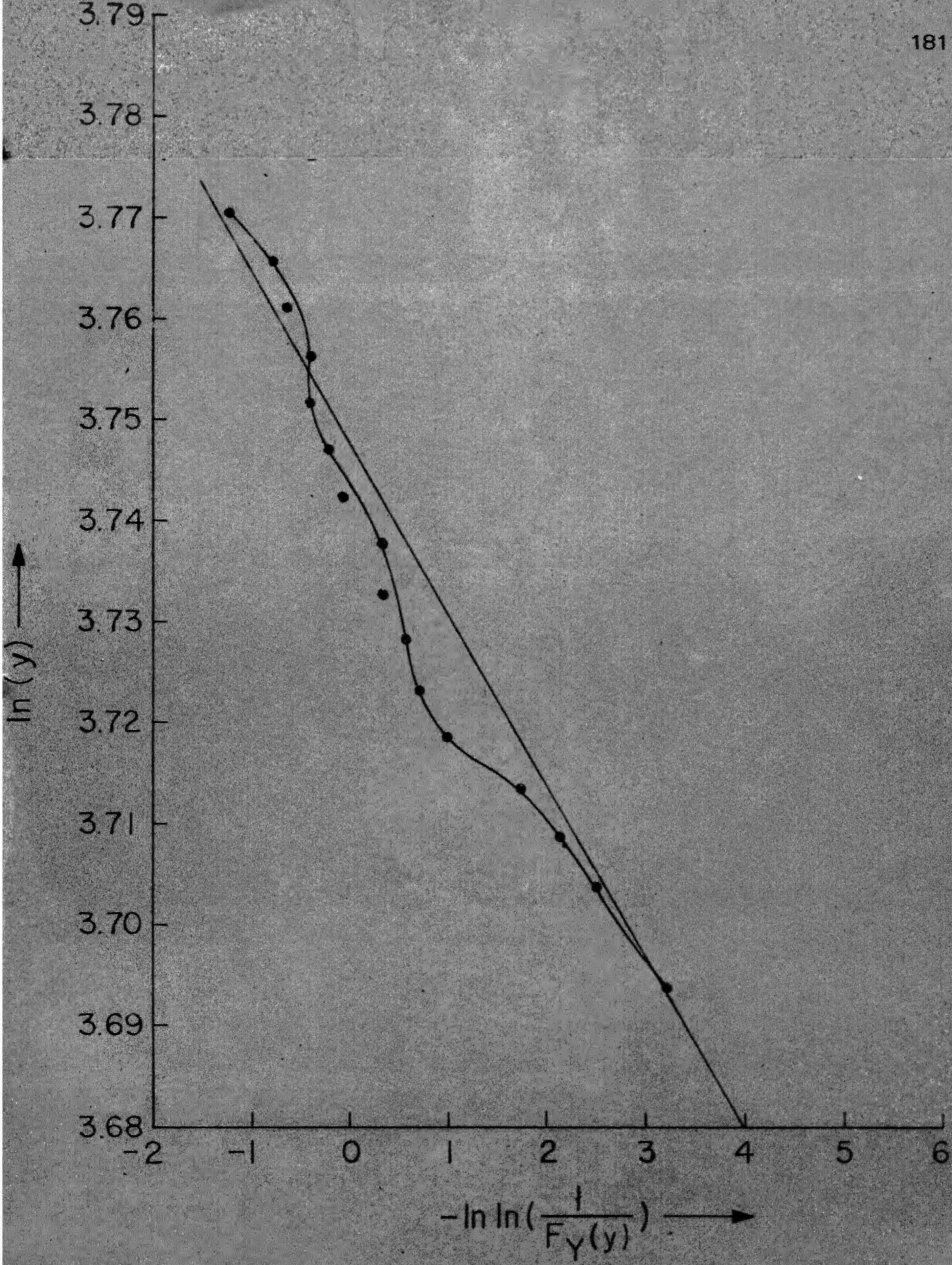


Fig. 6.9 Type II (maximum) distribution for yearly maximum temperature of Poona

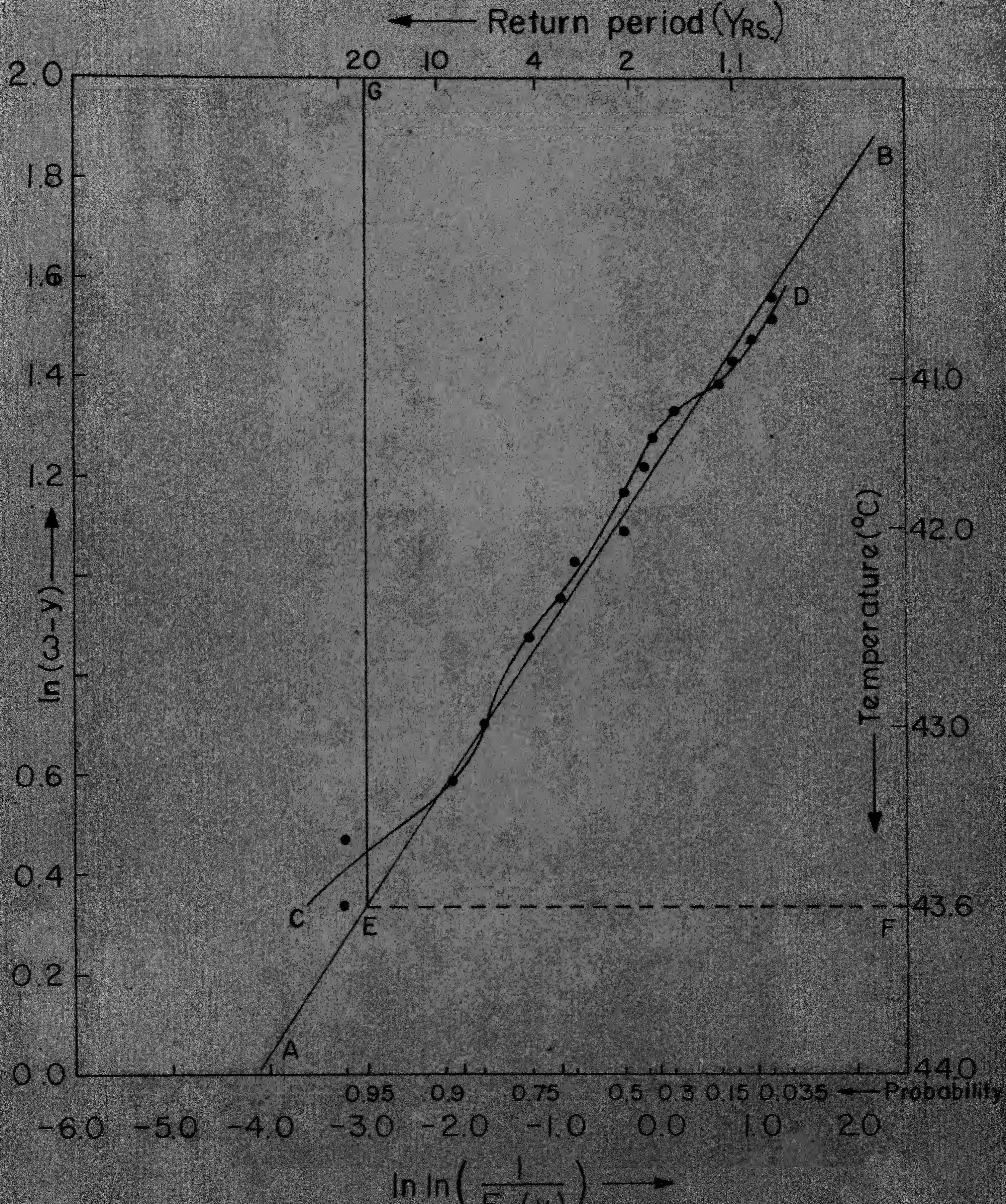


Fig. 6.10 Type III (maximum) distribution for yearly maximum temperature of Poona

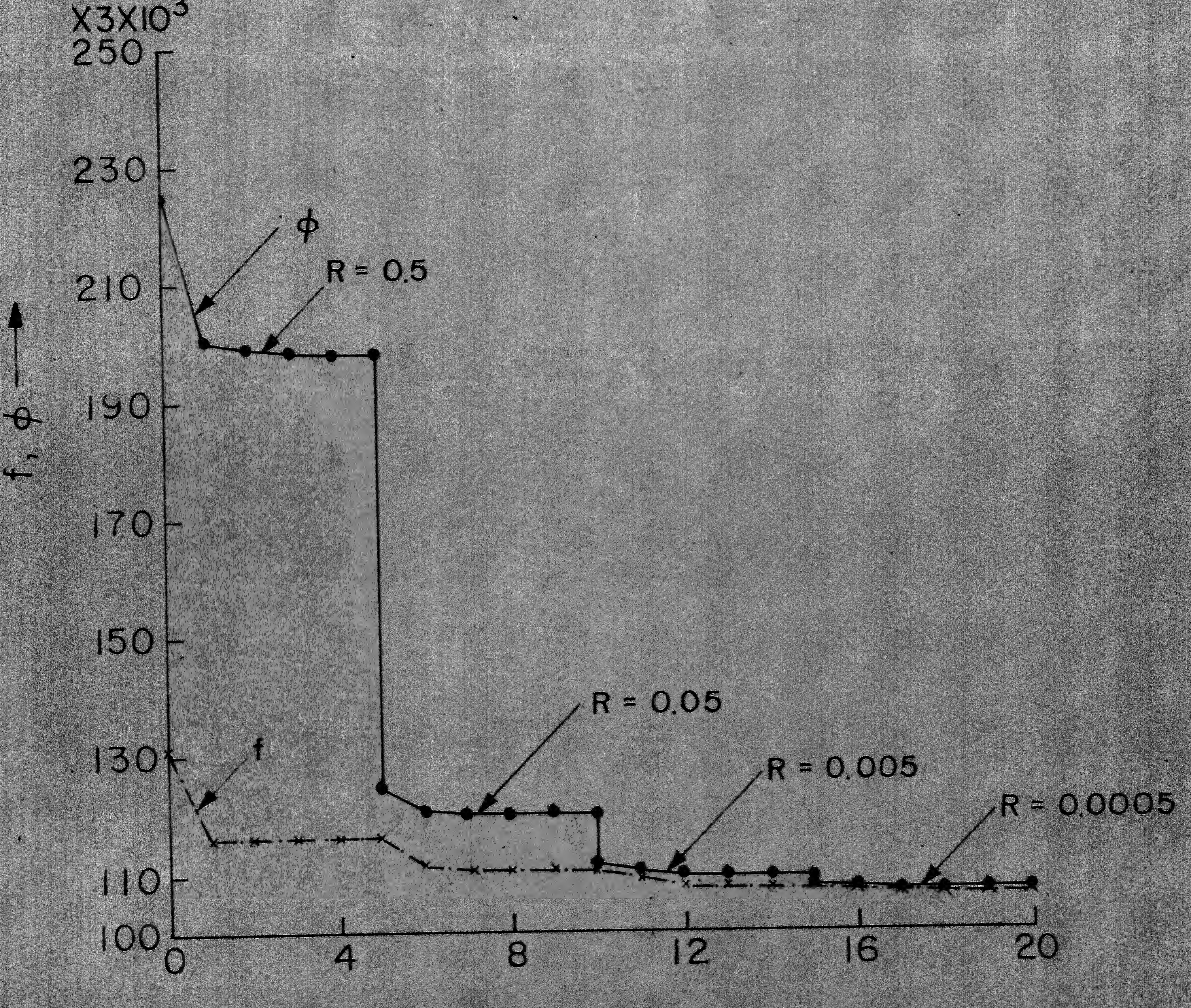


Fig. 6.11 Progress of optimization with cooling load as objective (Problem I)

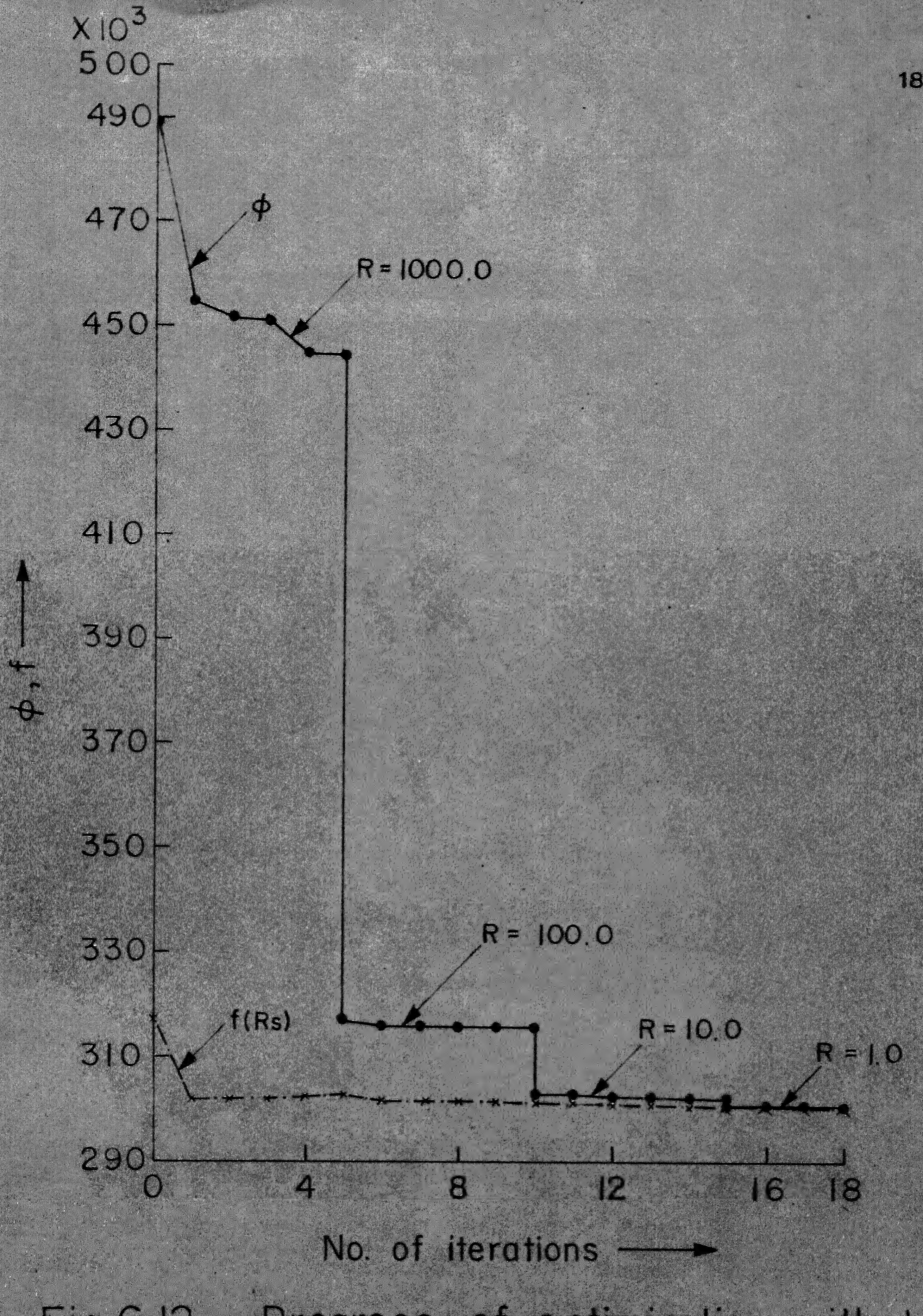


Fig. 6.12 Progress of optimization with total cost as objective (Problem 2)

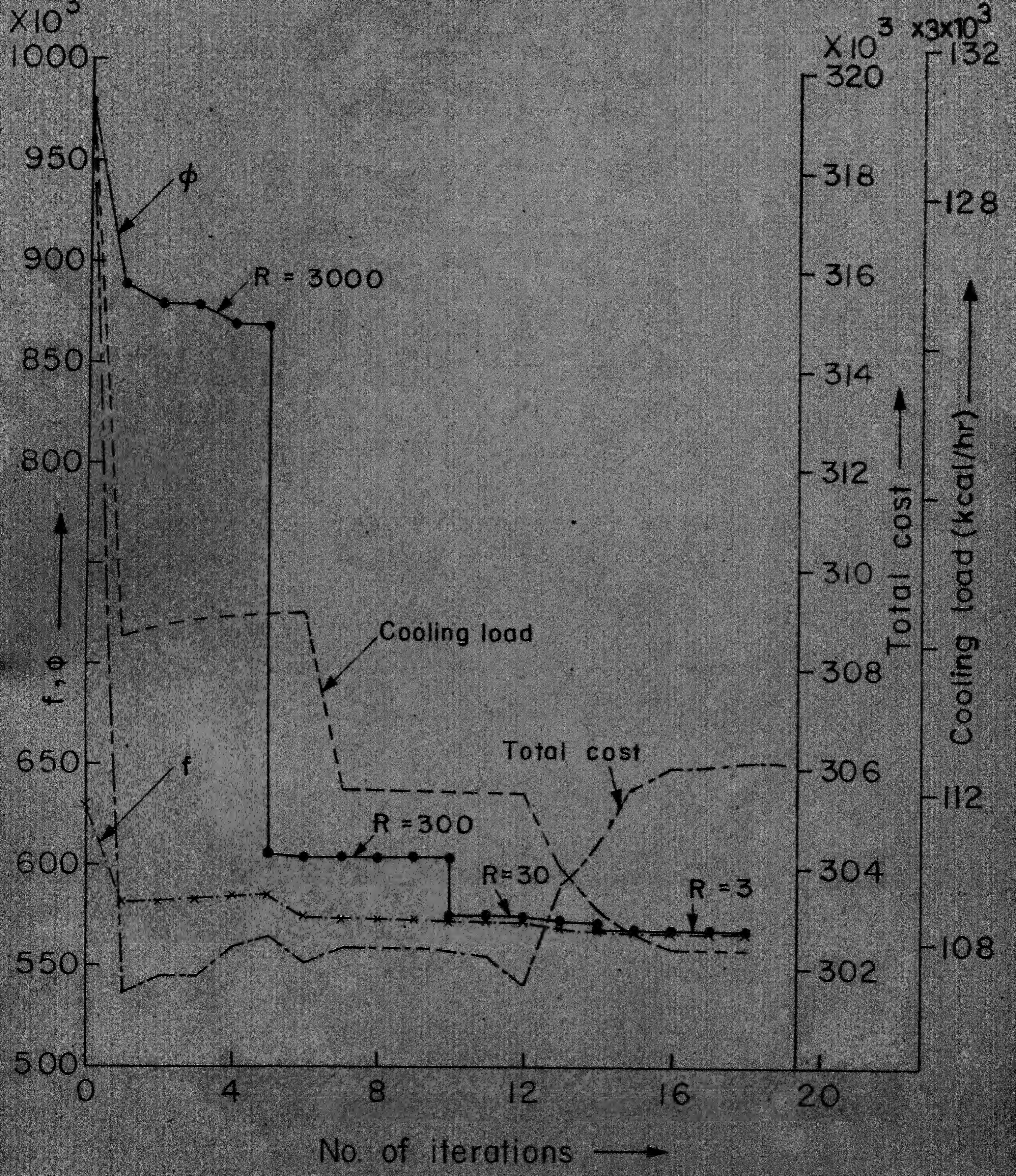


Fig. 6.13 Progress of optimization with weighted average of cost and cooling load as the objective problem 3

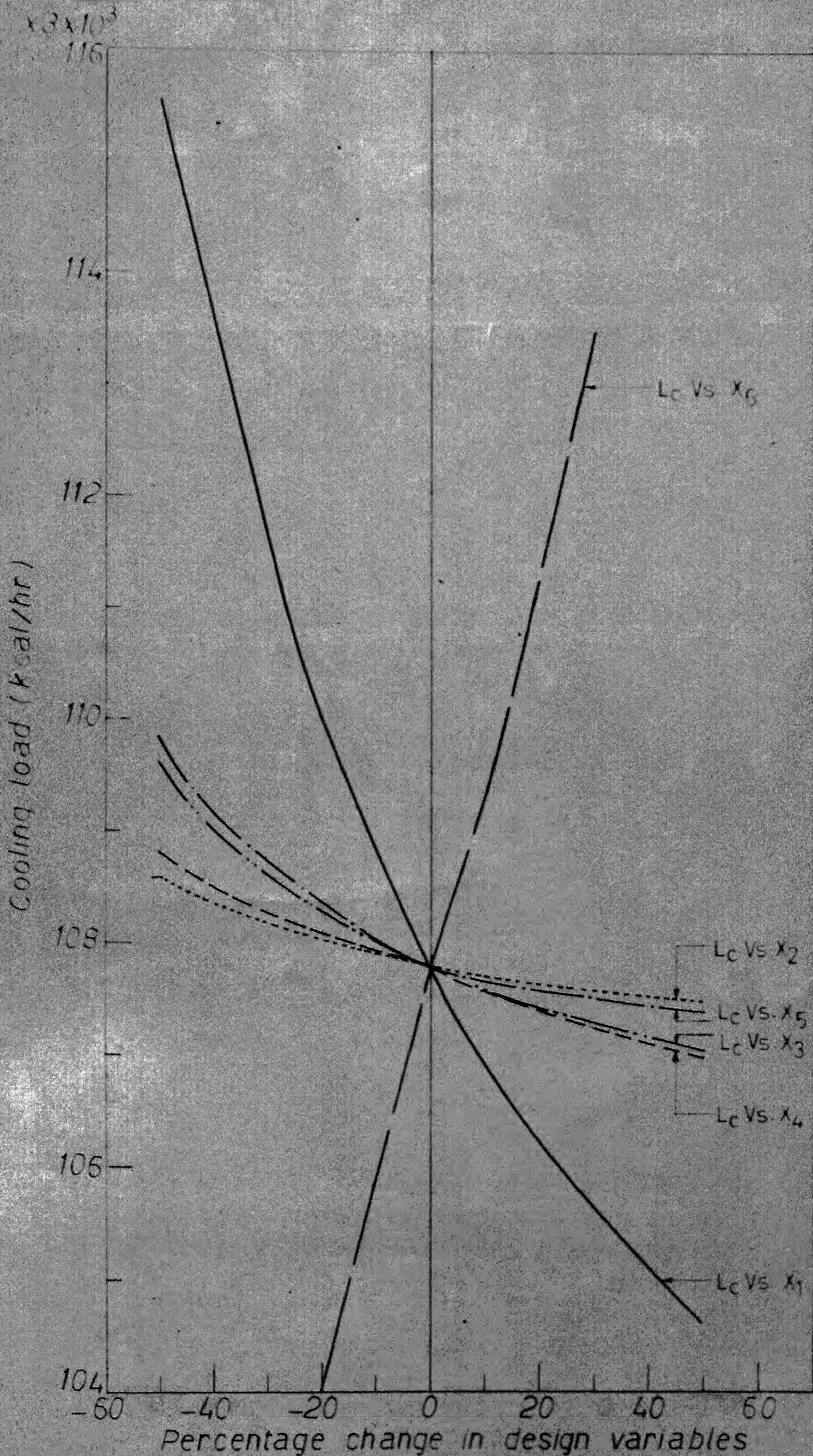


FIG. 6.14 SENSITIVITY ANALYSIS OF COOLING LOAD

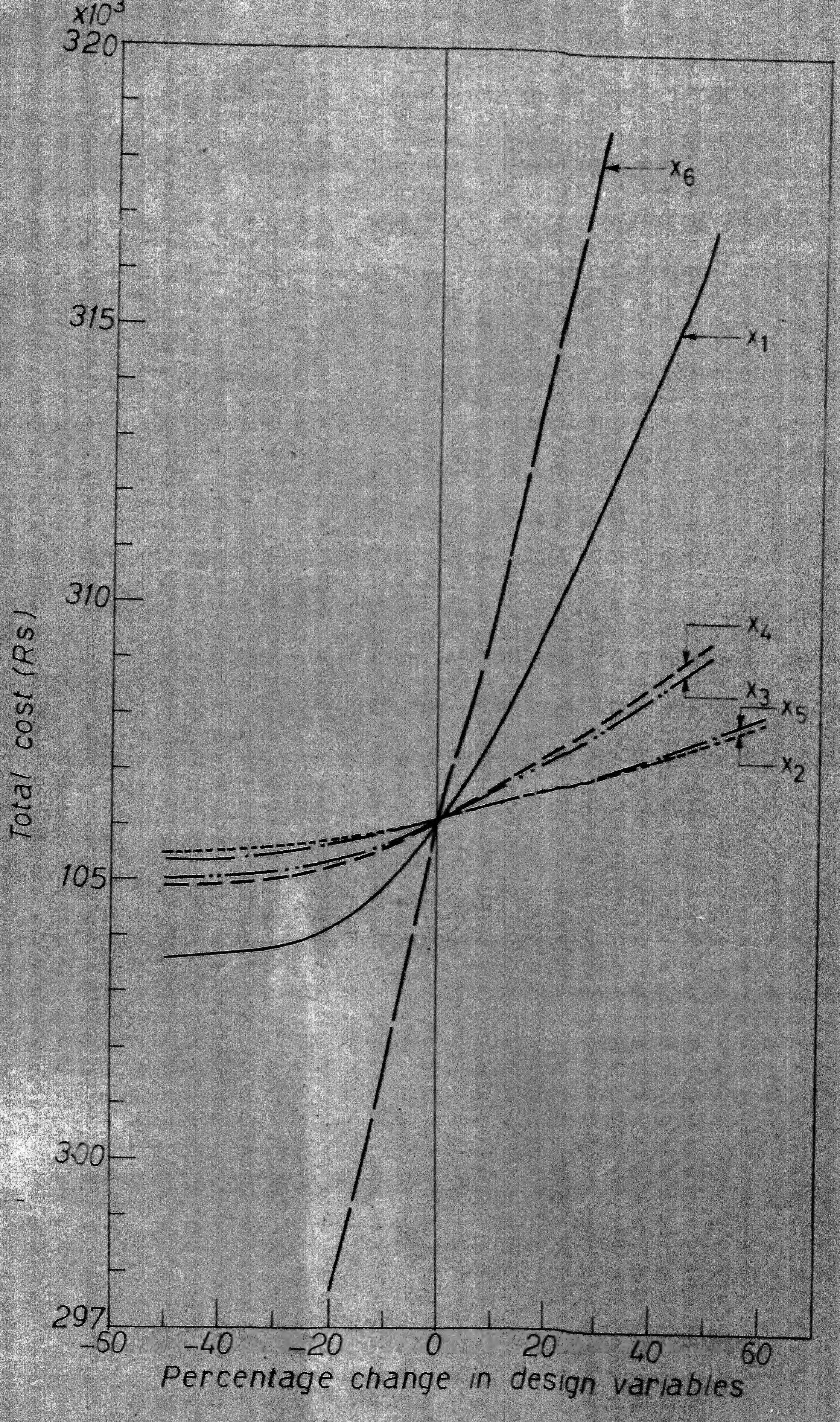


FIG. 6.15 SENSITIVITY ANALYSIS OF TOTAL COST.

CHAPTER 7

CONCLUSION AND RECOMMENDATIONS

7.1 Conclusions

(i) The optimum design of refrigerated warehouses and air-conditioned buildings using multidimensional optimization techniques has been considered in this work. The solution procedures to be adopted according to deterministic and probabilistic design philosophy are indicated and illustrated. The designs are based on twelve critical days (instead of one used in conventional procedures) selected on the basis of maximum average solar-air temperature. The minimization of cooling load as well as total cost are considered in obtaining the numerical results. In the computer programme developed, a linear combination of these two objectives is also considered so that proper weights can be assigned to the two objectives depending on the requirements.

(ii) When the design of roof, which is a major source of heat transfer, is considered, it was observed that minimum possible thickness of roof with appropriate insulation thickness would be most economical.

(iii) From the results of the problems in which the brick thickness is considered as a design variable, it has

been found that the optimum point corresponds to minimum thickness of the wall with proper insulation thicknesses. However, from physical (strength) considerations one may have to use larger wall thicknesses.

(iv) The orientation of the building at any given location is not found to have an influence greater than 0.6% in cost and 1.6% in cooling load at the optimum design.

(v) When building envelope parameters are also in the control of the designer, it will be more economical to use an aspect ratio of 1.5 to 1.75 (1.5 from cost consideration and 1.75 from energy consideration) and larger heights to reduce the cost as well as cooling load. It is interesting to note that this conclusion agrees with the present trend of the design of warehouses where larger heights are used.

(vi) The effect of economics models on the minimum cost design has also been studied. Since different cost models influence the optimum design, one has to decide about the economics model before designing a refrigerated warehouse/air-conditioned building.

(vii) The use of random parameters like geometric parameters, material properties and environmental conditions such as temperature, solar radiation and wind velocity, in the optimum design of refrigerated warehouses is illustrated. This procedure assumes that the cooling load follows

normal distribution, which is justified from the central limit theorem. Further, this assumption simplifies the calculations. The procedure developed, however, is quite general and can be used even when the cooling load follows a distribution other than normal distribution. Although the results obtained according to probabilistic procedure are not substantially different from the ones obtained by using deterministic procedure, the probabilistic procedure is expected to be more realistic and rational.

(viii) Three types of extremal (asymptotic) distributions are fitted into the maximum hourly temperature and solar radiation data and type III distribution (for largest values) has been found to be most suitable for describing these phenomena. An optimum design procedure using extremal distributions is also presented and illustrated by considering the design of a refrigerated warehouse by using the yearly maximum temperature data for 25 years. As expected, the optimum designs obtained by using extremal distribution have been found to be more conservative from the point of view of both cooling load and total cost compared to deterministic design and probabilistic design based on normal distribution.

(ix) The use of a weighted combination of cooling and heating loads in the design of air-conditioned buildings is

demonstrated. It is found that the insulation thickness corresponding to roof is maximum when the total load is taken as the criterion function while it has a lesser value when cost is taken as the objective.

(x) Whenever the stored commodity is more sensitive to fluctuations in the inside conditions (and large penalty or risk is involved), the inside conditions can be restricted to be at the desired levels in probabilistic design procedures.

(xi) The interior penalty function method has been found to be quite satisfactory for the optimum design of thermal systems. It has been observed that the largest decrease in the objective occurs in the very first iteration in all the cases. Hence if one is interested in finding only near-optimal solutions (rather than the exact optimum solutions), the process can be terminated after five or six iterations only thereby saving a substantial amount of computer time.

(xii) The results of sensitivity analyses conducted about the optimum design points are expected to be useful whenever the designer is required to round-off the design variables to the nearest available values. Further this information would be useful in identifying the lesser sensitive design variables so that the designer need not consider them while dealing with the design of similar systems.

7.2 Recommendations for Future Work

(i) The design of equipment* can also be considered along with that of building in the optimization procedure.

(ii) The analysis procedure used in the calculation of cooling load can be refined by using a more accurate method like matrix method.

(iii) The efficiency of other optimization methods like the method of feasible directions, geometric programming and dynamic programming can be studied.

(iv) The availability of discrete values of insulation thicknesses can be directly considered by using integer non-linear programming techniques.

(v) The optimum design of thermal systems using the concept of return period can also be studied.

(vi) The concept of overall reliability of the building system can also be incorporated with the optimum design procedure.

(vii) The accoustical considerations can also be incorporated along with thermal considerations in the optimum design of air-conditioned buildings.

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*Compressor, condenser, evaporator, ducts, piping etc.

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APPENDIX A

HEAT TRANSFER THROUGH A HOMOGENEOUS WALL

In this appendix the periodic transfer of heat through a wall made up of a single homogeneous material is considered. Figure A.1 shows the schematic problem. In this analysis it is assumed that (i) the heat flow is one dimensional (along x-direction), (ii) the wall is homogeneous with constant material properties, (iii) the surface coefficients h_i and h_o are constant, (iv) the solar absorptivity of the outside surface is independent of the angle of incidence and is constant, (v) the variation of outside air temperature t_o and incident solar radiation I are periodic, and (vi) the internal thermal environment is constant.

At any location within the wall, the one dimensional conduction equation can be written as

$$\frac{\partial t}{\partial \tau} = \alpha \frac{\partial^2 t}{\partial x^2} \qquad \text{(A.1)}$$

where

t = wall temperature at time τ and distance x,

α = thermal diffusivity = $\frac{K}{\rho C}$,

K = thermal conductivity of the wall material,

ρ = wall density and

C = specific heat for the wall material.

Equation (A.1) can be solved by using the two boundary conditions: (i) At the inside surface,

$$q_i = -K\left(\frac{\partial t}{\partial x}\right)_{x=L} = h_i(t_{w,i} - t_i) \tag{A.2}$$

where h_i = combined convection and radiation heat transfer coefficient for the inside surface of wall, $t_{w,i}$ = temperature of the inside surface of the wall and t_i = inside air temperature, and at the outside surface,

$$q_o = -K\left(\frac{\partial t}{\partial x}\right)_{x=0} = h_o(t_e - t_{w,o}) \tag{A.3}$$

where h_o = combined convection and radiation heat transfer coefficient for the outside surface of the wall, $t_{w,o}$ = temperature of the outside surface of the wall and t_e = solar-air temperature given by

$$t_e = t_o + \frac{aI}{h_o} \tag{A.4}$$

which can also be expressed as

$$t_e = t_{e,m} + t_{e,1}\cos(\omega_1\tau - \psi_1) + t_{e,2}\cos(\omega_2\tau - \psi_2) + \dots \tag{A.5}$$

where $t_{e,m}$ = 24-hour mean value of solar-air temperature and $t_{e,n}$ = harmonic coefficients ($n = 1, 2, \dots$). The complete solution of Eq. (A.1) has been given by Alford, Ryan and Urban [3]. The temperature of the inside-wall surface $t_{w,i}$ may be expressed as

$$t_{w,i} = t_i + \frac{1}{h_i}[U(t_{e,m} - t_i) + V_1 t_{e,1} \cos(\omega_1 \tau - \psi_1 - \phi_1) +$$

$$V_2 t_{e,2} \cos(\omega_2 \tau - \psi_2 - \phi_2) + \ldots] \tag{A.6}$$

where the overall heat transfer coefficient U is given by

$$U = \frac{1}{\frac{1}{h_i} + \frac{L}{K} + \frac{1}{h_o}} \tag{A.7}$$

and

$$t_{e,m} = \frac{1}{24} \int_o^{24} t_e \, d\tau \tag{A.8}$$

Since the values of t_e can only be computed at hourly intervals from the available data, Eq. (A.8) is taken as

$$t_{e,m} = \frac{1}{24}[t_{e1} + t_{e2} + \ldots + t_{e24}] \tag{A.9}$$

The expressions for V_n, Y_n and Z_n are given by

$$V_n = \frac{h_o h_i}{S_n K \, (Y_n^2 + Z_n^2)^{1/2}} \tag{A.10}$$

where n = order of the harmonic in the Fourier series of Eq. (A.6),

$$S_n = (\frac{\omega_n}{2\alpha})^{1/2} \tag{A.11}$$

where ω_n is the angular velocity of the sinusoidal wave,

$$Y_n = \left(\frac{h_o h_i}{2S_n^2K^2} + 1\right) \cos S_nL \sinh S_nL + \left(\frac{h_o h_i}{2S_n^2K^2} - 1\right) \sin S_nL \cos S_nL$$

$$+ \frac{(h_o + h_i)}{S_nK} \cos S_nL \cosh S_nL \qquad (A.12)$$

$$Z_n = \left(\frac{h_o h_i}{2S_n^2K^2} + 1\right) \sin S_nL \cosh S_nL - \left(\frac{h_o h_i}{2S_n^2K^2} - 1\right) \cos S_nL$$

$$\sinh S_nL + \left(\frac{h_o + h_i}{S_nK}\right) \sin S_nL \sinh S_nL \qquad (A.13)$$

and

$$\phi_n = \tan^{-1} \frac{Z_n}{Y_n} \qquad (A.14)$$

In Eq. (A.14), $\sin\phi_n$ has the sign of Z_n and $\cos\phi_n$ has the sign of Y_n.

Again

$$t_{e,n} = (M_n^2 + N_n^2)^{1/2} \qquad (A.15)$$

where

$$M_n = \frac{1}{12} \int_0^{24} t_e \cos \omega_n \tau \, d\tau \qquad (A.16)$$

$$N_n = \frac{1}{12} \int_0^{24} t_e \sin \omega_n \tau \, d\tau \qquad (A.17)$$

$\omega_1 = \pi/12$ radians per hour or 15° per hour, and $\omega_n = n\omega$.

In Eqs. (A.16) and (A.17), the integrations can be replaced by summations because only hourly values can be computed for the solar-air temperature. Hence

$$M_n = \frac{1}{12} \sum_{0}^{24} t_e \cos \omega_n \tau \qquad (A.18)$$

$$N_n = \frac{1}{12} \sum_{0}^{24} t_e \sin \omega_n \tau \qquad (A.19)$$

and

$$\psi_n = \tan^{-1} \frac{N_n}{M_n} \qquad (A.20)$$

In Eq. (A.20), the quadrant in which ψ_n lies is determined by the requirements that $\sin \psi_n$ must have the sign of N_n and $\cos \psi_n$ the sign of M_n.

The rate of heat transfer to the interior can be expressed as

$$q_i = h_i(t_{w,i} - t_i) \quad . \qquad (A.21)$$

By substituting Eq. (A.6) for $t_{w,i}$, Eq. (A.21) can be written as

$$q_i = U\{[t_{e,m} + \lambda_1 t_{e,1} \cos(\omega_1 \tau - \psi_1 - \phi_1) + \lambda_2 t_{e,2}(\omega_2 \tau - \psi_2 - \phi_2) + \ldots] - t_i\} \qquad (A.22)$$

where

$$\lambda_n = \frac{V_n}{U} \qquad (A.23)$$

Here λ_n is called the decrement factor and ϕ_n is the angular displacement or lag between a harmonic of the solar-air temperature and the same harmonic of the inside surface temperature.

Equivalent temperature difference:

The form of Eq. (A.22) is interesting. Although q_i may be continuously changing, it may be calculated by multiplying the overall heat transfer coefficient U for steady state heat transmission by an equivalent temperature difference which accounts for the periodic variation of solar-air temperature t_e and the heat storage characteristics of the wall. Equation (A.22) can be written as

$$q_i = U \Delta T_{EQ} \tag{A.24}$$

where

$$\Delta T_{EQ} = [\, t_{e,m} + \lambda_1 t_{e,1} \cos(\omega_1 \tau - \psi_1 - \phi_1) + \lambda_2 t_{e,2} \cos(\omega_2 \tau - \psi_2 - \phi_2) + \ldots \,] - t_i \tag{A.25}$$

where ΔT_{EQ} is known as the equivalent temperature difference.

In the present work, Eq. (A.22) is used to compute the heat gain through the homogeneous wall. The order of harmonics is taken as 2 because it was found that with 4 and 6 order harmonics, there is negligible difference in the heat transfer.

The flow chart for the heat gain through exterior walls and roof along with the other subroutines used in the computer programme is given in Figure A.2.

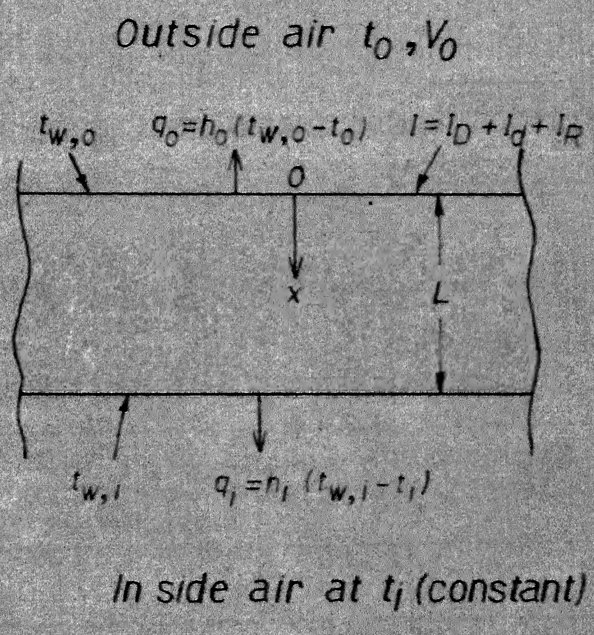
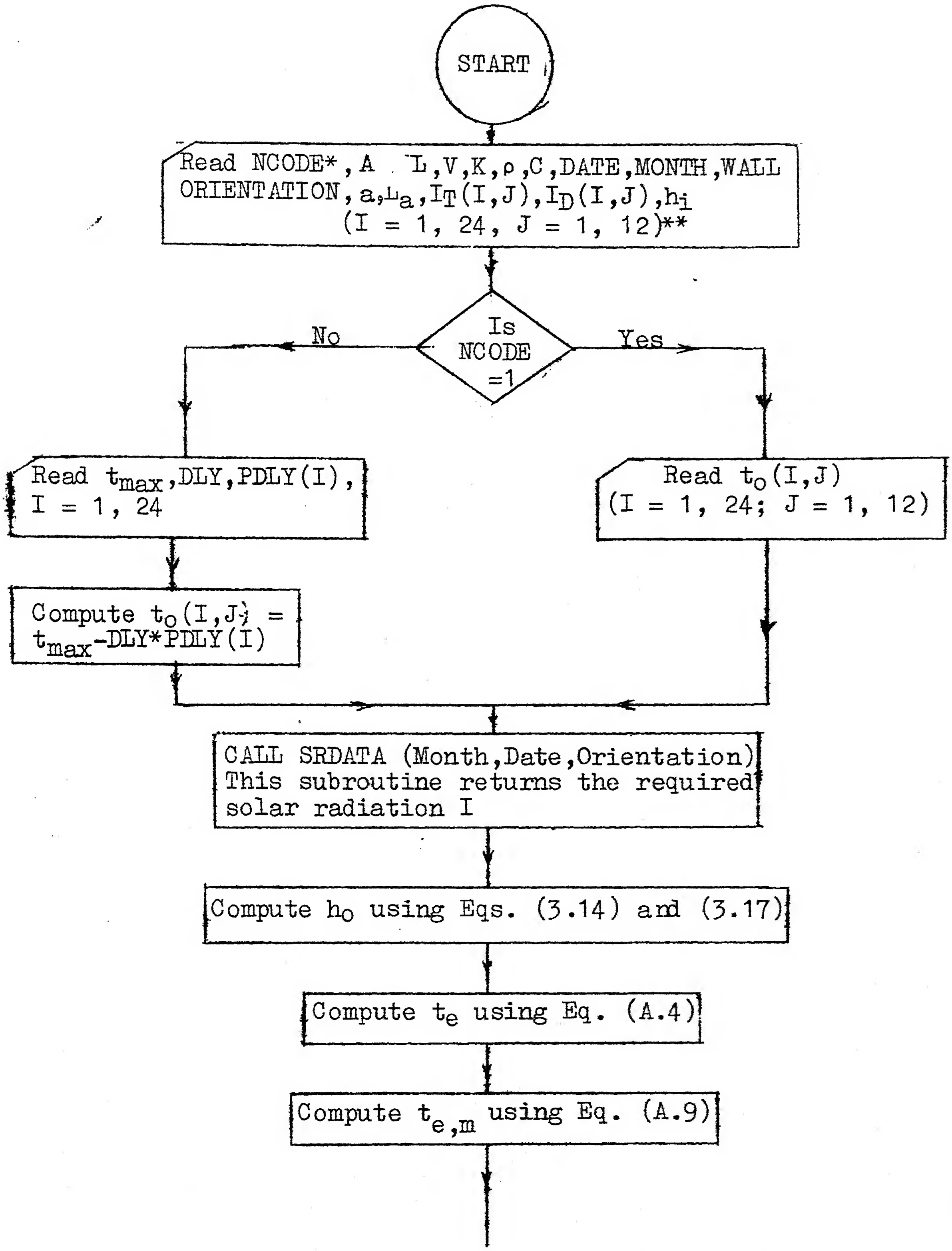
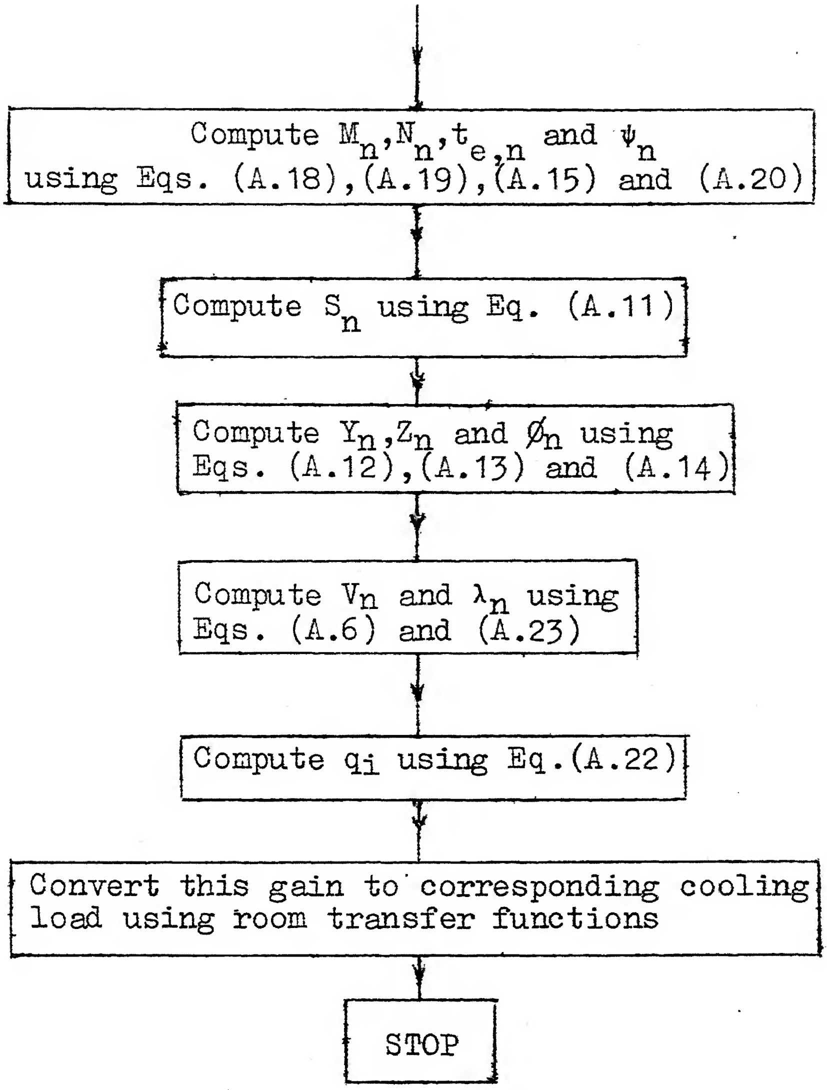


FIG. A.1 SCHEMATIC DIAGRAM FOR PERIODIC HEAT TRANSFER ANALYSIS


START
Read NCODE*, A L, V, K, ρ, C, DATE, MONTH, WALL ORIENTATION, a, La, IT(I,J), ID(I,J), hi (I = 1, 24, J = 1, 12)**
Is NCODE =1
No
Yes
Read tmax, DLY, PDLY(I), I = 1, 24
Compute to(I,J) = tmax-DLY*PDLY(I)
Read to(I,J) (I = 1, 24; J = 1, 12)
CALL SRDATA (Month, Date, Orientation) This subroutine returns the required solar radiation I
Compute ho using Eqs. (3.14) and (3.17)
Compute te using Eq. (A.4)
Compute te,m using Eq. (A.9)



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*If NCODE = 1 Hourly outdoor air temperature data is available

$\neq 1$ Hourly temperature data is to be computed using maximum outdoor air temperature t_{max}, daily range DLY and percentage daily range PDLY

**I = 1,...,24 (number of hours per day)

J = 1,...,12 (number of months per year)

DETAILS OF OTHER SUBROUTINES USE IN THE COMPUTER PROGRAMME

FTN	:	To compute cooling load through different external and internal sources and objective and Ø-function
FTNN	:	To evaluate maximum inside surface temperature
GLHT	:	To evaluate the heat gain through glass
GRADN	:	To evaluate the gradient of Ø-function using forward difference formula
MINIM4	:	To implement the Davidon-Fletcher-Powell variable metric method of unconstrained minimization
SLOPE2	:	To evaluate the slope of Ø-function
SRDATA	:	To compute solar radiation incident on any wall or roof at any hour of the day
STEPX3	:	To evaluate step length using cubic interpolation technique of one dimensional minimization.

FIGURE A.2 FLOW CHART FOR THE HEAT TRANSFER THROUGH WALLS AND ROOF

APPENDIX B

DATA USED IN COMPUTATIONS OF HEAT GAIN*

TABLE B.1

VALUES OF EQUATION OF TIME FOR DIFFERENT MONTHS

Month/Date		Equation of time (min : sec.)	Month/Date		Equation of time (min : sec.)
January	21	- 11 : 18	July	21	- 6 : 25
February	21	- 13 : 28	August	21	- 1 : 18
March	21	- 7 : 19	September	21	+ 7 : 30
April	21	+ 0 : 08	October	21	+ 15 : 06
May	21	+ 3 : 32	November	21	+ 13 : 55
June	21	- 1 : 48	December	21	+ 1 : 32

TABLE B.2

VALUES OF DECLINATION FOR DIFFERENT MONTHS

Month/Date		Declination (degree)	Month/Date		Declination (degree)
January	21	- 20.00	July	21	+ 20.60
February	21	- 10.80	August	21	+ 12.30
March	21	0.00	September	21	0.00
April	21	+ 11.60	October	21	- 10.50
May	21	+ 20.00	November	21	- 19.80
June	21	+ 23.45	December	21	- 23.45

*All the data given in this appendix is in F.P.S. units. This data has been converted with M.K.S. units in actual computations.

TABLE B.3

AVERAGE AIR CHANGES PER 24 HOURS FOR STORAGE ROOMS DUE TO DOOR OPENINGS AND INFILTRATION

Volume cu. ft.	Number of air changes per 24 hours
200	44.0
300	34.5
400	29.5
500	26.0
600	23.0
800	20.0
1000	17.5
1500	14.0
2000	12.0
3000	9.5
4000	8.2
5000	7.2
6000	6.5
8000	5.5
10000	4.9
15000	3.9
20000	3.5
25000	3.0
30000	2.7
40000	2.3
50000	2.0
75000	1.6
100000	1.4

Note: For heavy usage multiply the above values by 2.0.

For long storage multiply the above values by 0.6.

TABLE B.4

RATE OF HEAT GAIN FROM OCCUPANTS OF CONDITIONED SPACES

Degree of activity	Typical application	Total heat adults male (BTU/hr)	Total heat adjusted (BTU/hr)	Sensible heat (BTU/hr)	Latent heat (BTU/hr)
Seated at rest	Theatre - Matinee	390	330	225	105
	Theatre - Evening	390	350	245	105
Seated at very light work	Office, hotels, apartments	450	400	245	155
Moderately active office work	Office, hotels, apartments	475	450	250	200
Standing, light work; or walking slowly	Department store, retail store, dime store	550	500	250	250
Walking, seated, standing, walking slowly	Drug store, bank	550	500	250	250
Sedentary work	Restaurant	490	550	275	275
Light bench work	Factory	800	750	275	475
Moderate dancing	Dance hall	900	850	305	545
Walking at 3 mph; moderately heavy work	Factory	1000	1000	375	625
Bowling, heavy work	Bowling alley, factory	1500	1450	580	870

TABLE B.5

DATA OF FOOD PRODUCT

Product	Average freezing point (°F)	Percent water contents	Specific heat (BTU/lb °F)		Latent heat of fusion (BTU/lb)	Heat of respiration (BTU/24 hr/ton) at temperature indicated	
			Above freezing	Below freezing			
Potatoes (white)	28.9	77.8	0.82	0.43	111	40°F	1300-1800
Potatoes (sweet)	28.5	68.5	0.75	0.40	97	40°F	1710
Onions	30.1	87.5	0.90	0.46	124	32°C	700-1100
Tomatoes (green)	30.4	94.7	0.95	0.48	134	60°F	1260
Tomatoes (ripening)	30.4	94.1	0.95	0.48	134	40°F	1900
Apples	28.4	84.1	0.86	0.45	121	32°F	830
Oranges	28.0	87.2	0.90	0.46	124	32°F	765
Fish (frozen)	28.0	70.0	0.76	0.41	101		

APPENDIX C

DATA RELATED TO THE CONVERSION OF HEAT GAIN TO COOLING LOAD

TABLE C.1

CONVECTIVE AND RADIANT HEAT GAIN TO COOLING LOAD

Heat gain component	% radiant	% convective	% latent
Heat transfer through transparent surfaces (without inside shading) by solar radiation	100	0	0
Heat transfer through transparent surfaces (with inside shading solar radiation)	58	42	0
Heat transfer through transparent surfaces (with or without inside shading) by air to air convection and conduction	0	100	0
Heat addition by fluorescent lights	50	50	0
Heat addition by incandescent lights	80	20	0
Energy addition by people	40	20	40
Energy addition by machinery or appliances*	-	-	-
Energy addition by infiltration and ventilation	0	Convective and latent account for 100%	
Heat transfer through interior partitions or exterior walls and roofs	60	40	0

*Percentage will vary according to the device being considered. The sensible portion could vary from 20 to 80%, radiant (and 80 to 20% convective) depending on its surface temperature.

TABLE C.2

COEFFICIENTS OF ROOM TRANSFER FUNCTION[a]

Heat gain component	C[b]	v_o	v_1	v_2	v_3
			Dimensionless		
Solar heat gain[c] through glass with no interior shading device	L	0.2727	-0.3400	0.1169	-0.0064
	M	0.2217	-0.3354	0.1448	-0.0128
	H	0.2155	-0.3712	0.1790	-0.0166
Construction heat gain through roofs, exterior walls, partitions and doors and windows with blinds or drapes	L	0.6982	-1.2017	0.6617	-0.1150
	M	0.7108	-1.4456	0.9639	-0.2108
	H	0.7055	-1.5668	1.1378	-0.2698
Heat generated by the light[d]	L	0.3178	-0.4507	0.2089	-0.0328
	M	0.2605	-0.4662	0.2819	-0.0579
	H	0.2430	-0.5085	0.3547	-0.0825
Heat generated by equipment and people and dissipated by radiation	L	0.3251	-0.4267	0.1524	-0.0076
	M	0.2574	-0.4038	0.1830	-0.0183
	H	0.2503	-0.4446	0.2255	-0.0245
Heat generated by equipment and people and dissipated by convection and sensible heat gain by ventilation and infiltration	These heat gain components appear as cooling load on air-conditioning unit without delay. Thus $v_o = 1.0, \quad v_1 = v_2 = v_3 = w_1 = w_2 = w_3 = 0$				

Continued...

Table C.2 (Continued)

Heat gain component	C^b	v_o	v_1	v_2	v_3
			Dimensionless		
The w coefficients[e]		w_o	w_1	w_2	w_3
	L	1.000	-1.8260	1.0697	-0.2005
	M	1.000	-2.1092	1.4606	-0.3331
	H	1.000	-2.2908	1.7252	-0.4277

[a]The coefficient of heat transfer function for the case when all the heat gain energy appears eventually as cooling load.

[b]The letters L, M and H denote the following room construction:
L = Light construction; M = Medium construction; H = Heavy construction.

[c]The coefficients of transfer function that relate the room cooling load to the solar heat gain through glass depend on where the solar energy is absorbed.

[d]If the ventilating air is exhausted through the space above the ceiling, it removes some of the heat from the lights before it enters the room. This heat is a load on the air-conditioning plant if the air is recirculated, even though it is not part of the heat gain of the room. The percent of heat gain appearing in the room of lighting fixture its mounting and exhaust air flow.

[e]The w_i coefficients (of the denominator) apply to all components of input except for the convection part of heat gain from people and equipment.